# Enumeration for MSO-Queries on Compressed Trees

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A node-labelled tree $T$ with node set $V_T$.

**Task**:

Enumerate $q(T) = \{S \subseteq V_T \mid T \models q(S)\}$.

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Problem investigated in:

Bagan, CSL 2006
Courcelle, Discrete Applied Mathematics 2009
Kazana, Segoufin, ACM Trans. Comput. Log. 2013
Niewerth, LICS 2018
Amarilli, Bourhis, Mengel, Niewerth, PODS 2019

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A compressed node-labelled tree $T$ with node set $V_T$.

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# The Compression Scheme: Straight-Line Programs (SLPs)

A lossless compression scheme most famous for strings.

Practically highly relevant: Fast compressors with good compression.

Lots of practical and theoretical work on SLPs.

Have been applied in database theory.

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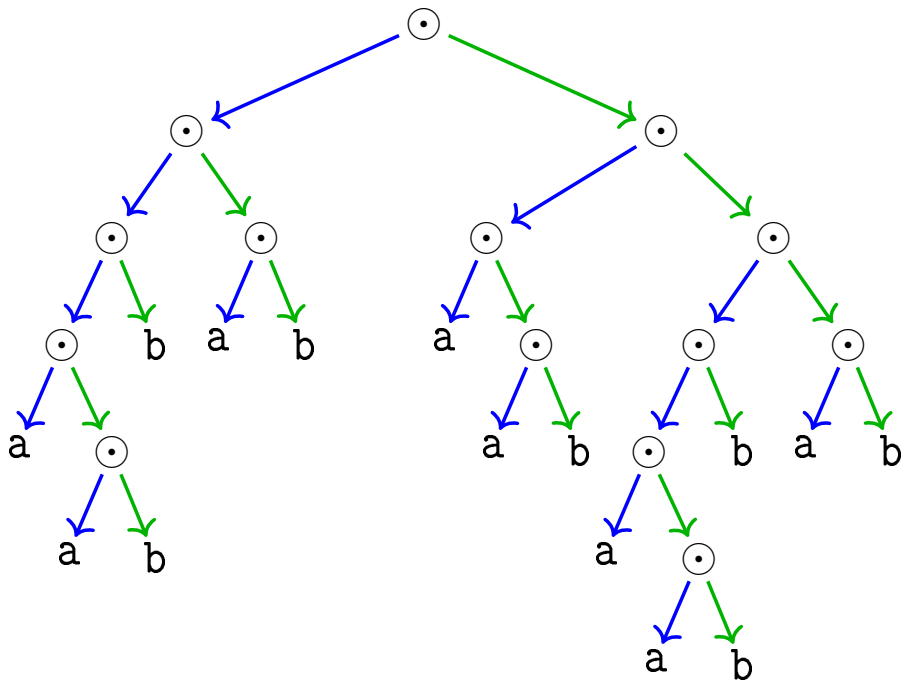


Formula as syntax tree.

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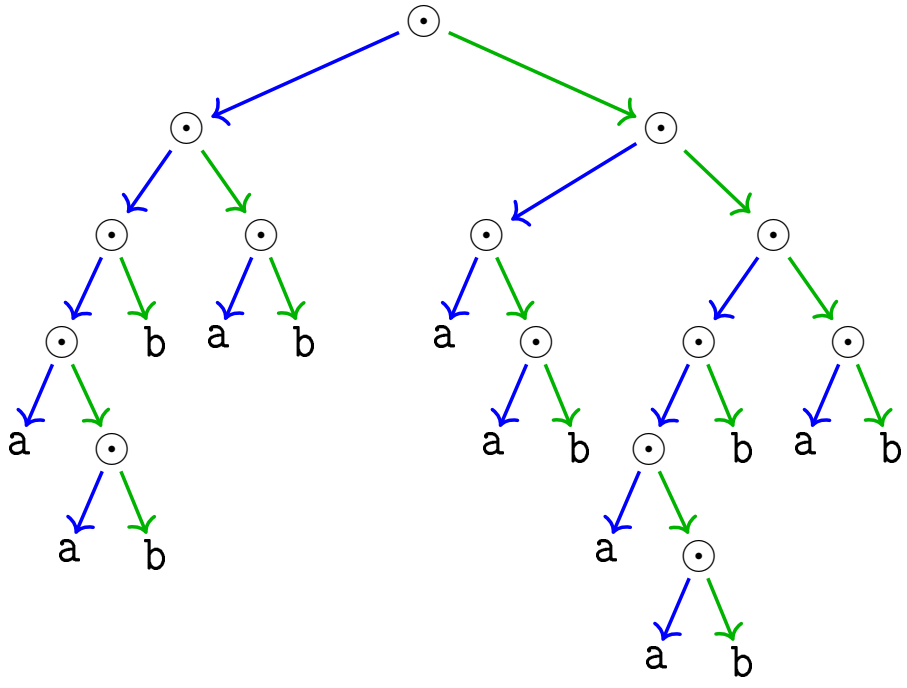


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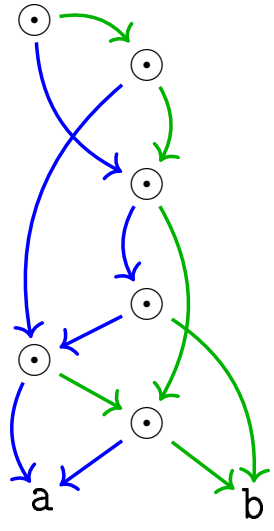


DAG-folding.
(SLP)

# SLPs for Trees and Forests

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Forest Algebra (Bojańczyk, Walukiewicz, 2008):

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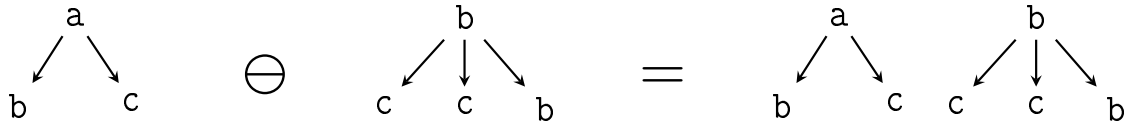


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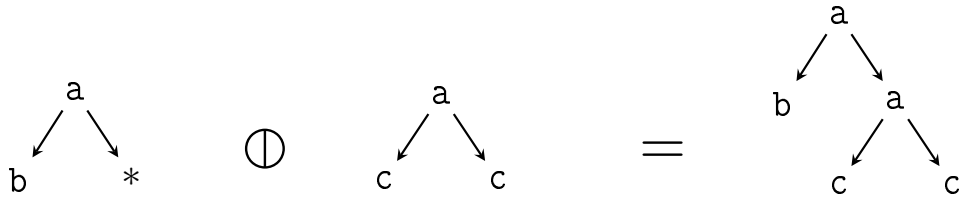

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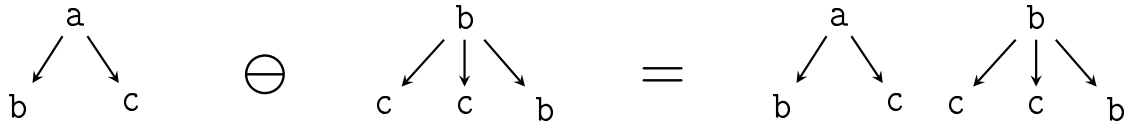


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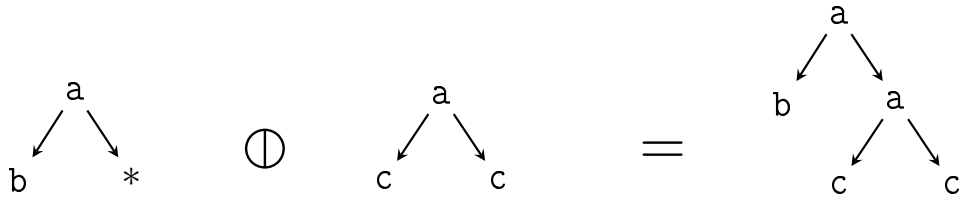


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$x$ represents a single $x$-labelled node and $x_*$ represents $\overset{x}{\downarrow}{}^{*}$.

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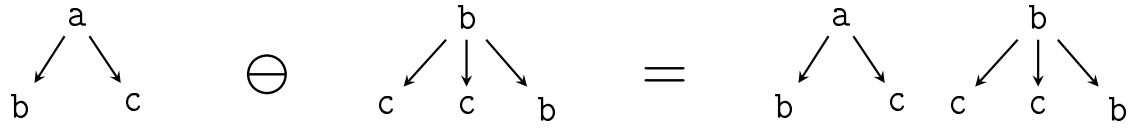


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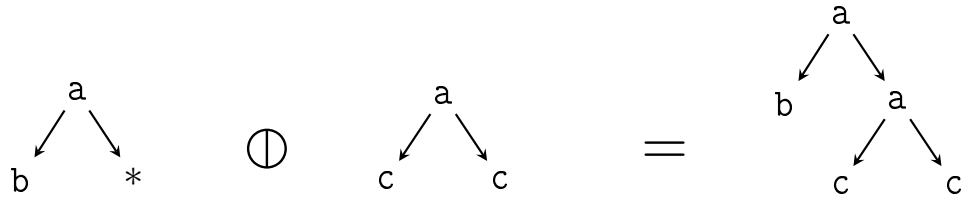


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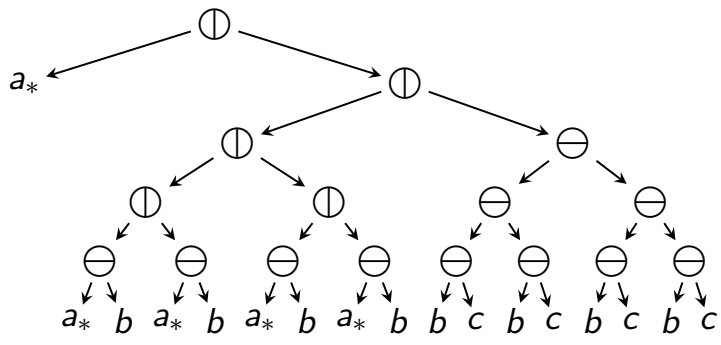
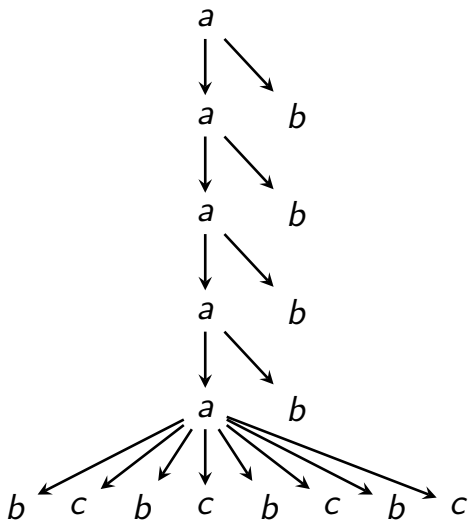
Example formula:



# Forest Straight-Line Programs (FSLPs)

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Gascón, Lohrey, Maneth, Reh, Sieber, 2020

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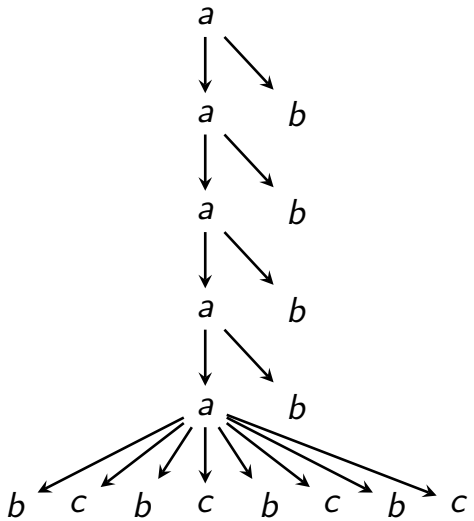


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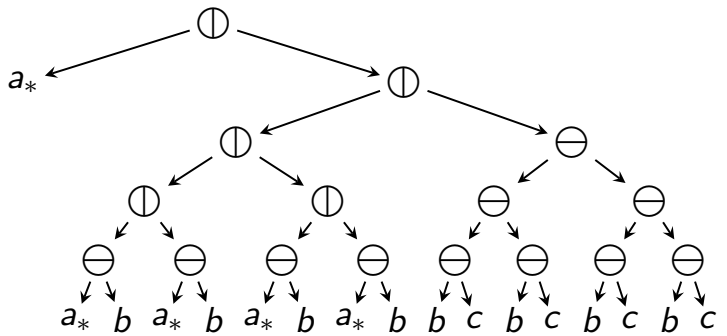


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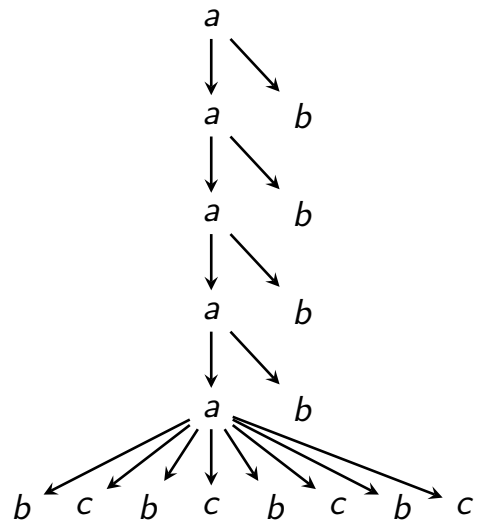


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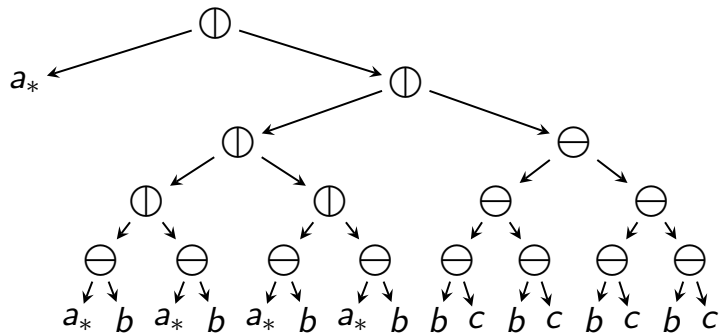


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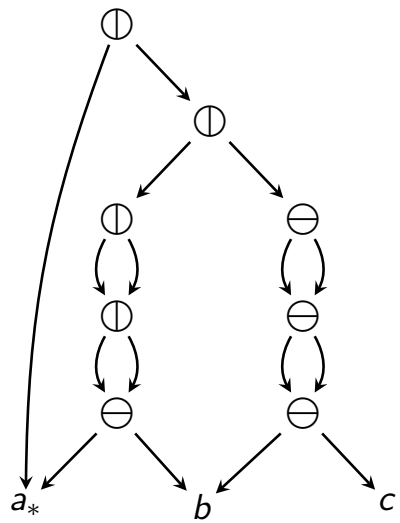


A forest-SLP for $F$.

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# Enumeration of MSO-Queries over SLP-Compressed Forests

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An FSLP $D$ that represents a node-labelled unranked forest $F$.

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Practical relevance:

An experimental study shows that in linear time, we can compress a corpus of typical XML trees by FSLPs to approximately 3% of its actual size.

[Lohrey, Maneth, Mennicke, 2013]

# Reduction to dBUTA Enumeration on DAG-Foldings

By known techniques, we reduce the problem to:

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Deterministic bottom-up tree automaton $\mathcal{A}$.

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**Task**:

After preprocessing $O(|D|)$, enumerate

$$\text{select}(\mathcal{A}, T) := \{S \subseteq V_{\text{leafs}} \mid \text{mark}(T, S) \in L(\mathcal{A})\}$$

with output linear delay.

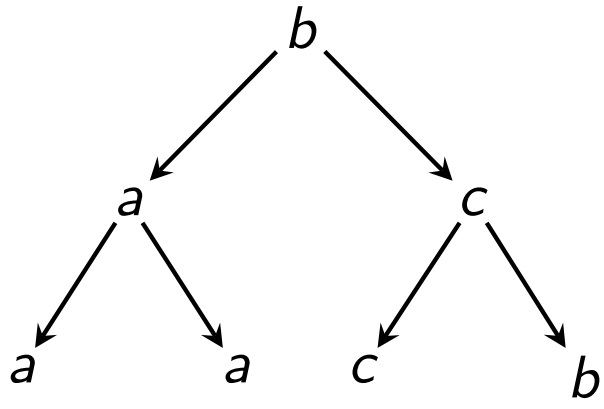
$\text{mark}(T, S)$: Tree $T$ with leaves from $S$ being marked.

# Bagan's Algorithm

**Theorem** — Bagan 2006

Given leaf-selecting dBUTA $\mathcal{B}$ and a binary node-labelled tree $T$, after a preprocessing in time $O(|T|)$, we can enumerate select$(\mathcal{A}, T)$ with output linear delay.

# Deterministic Bottom-Up Tree Automata



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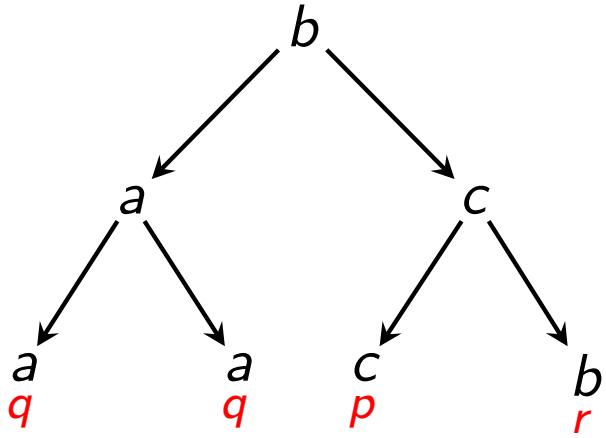


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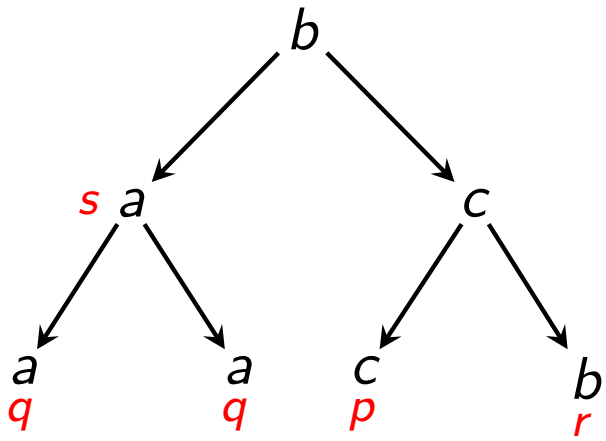


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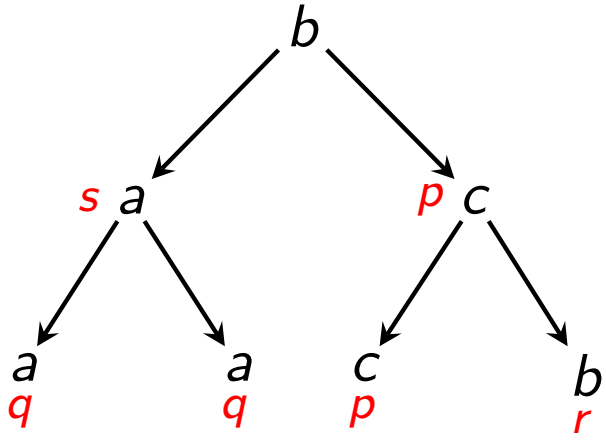


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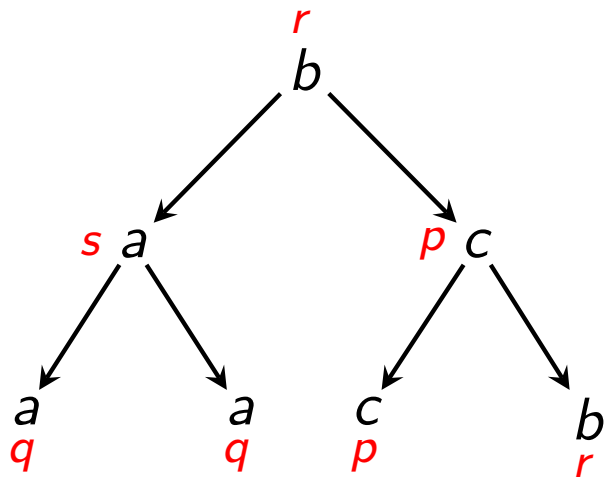


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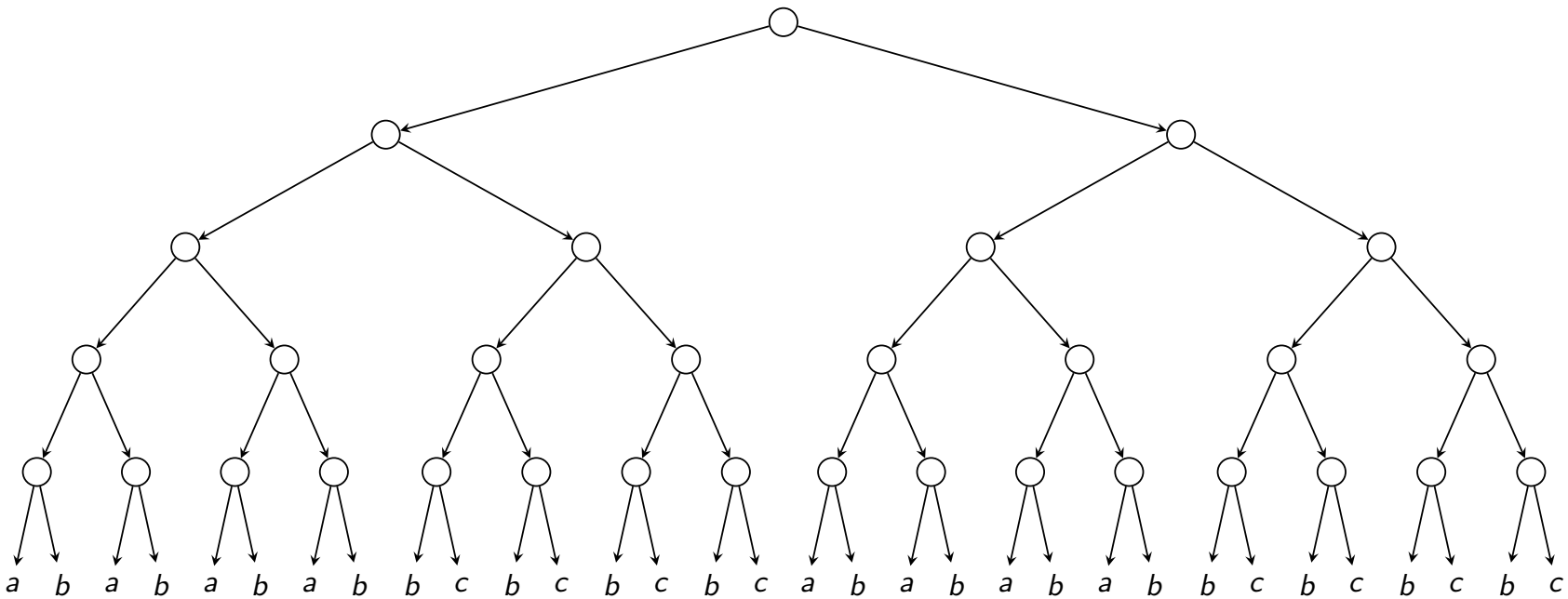
Leaf-labelled tree $T$:



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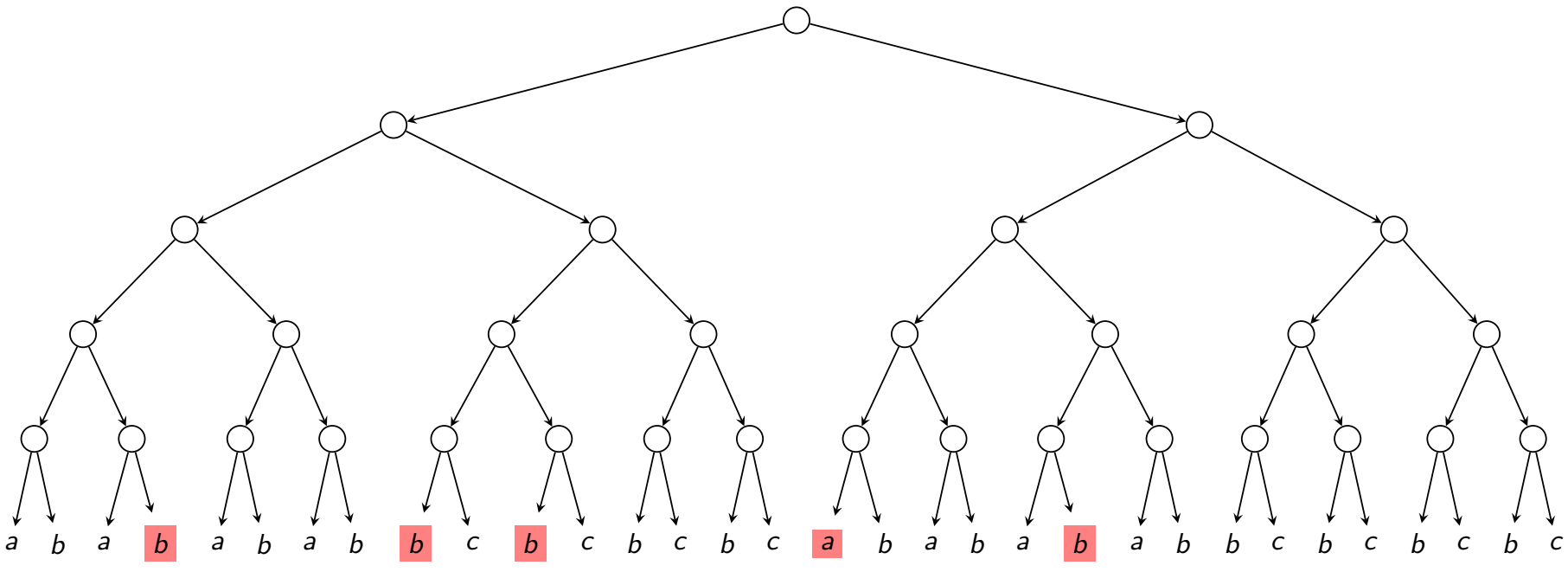
Marked tree mark($T, S$) for leaf-set $S$:



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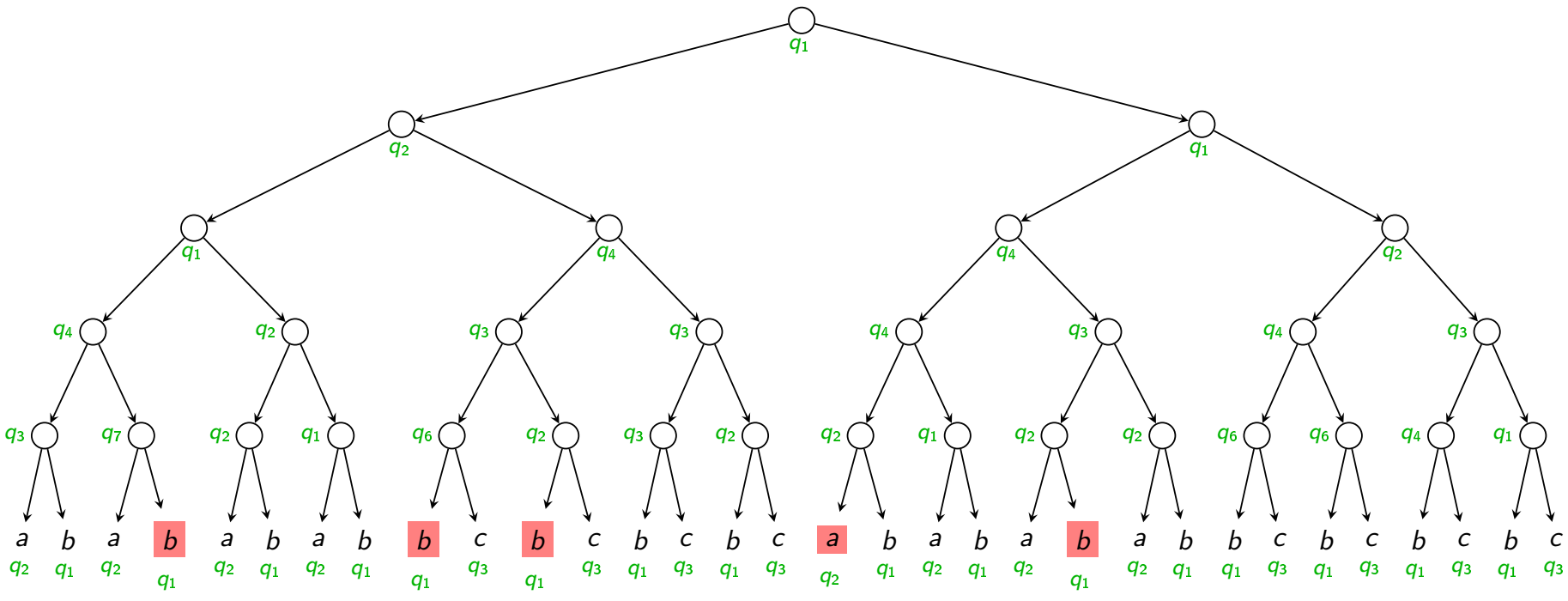
Run on mark($T, S$):



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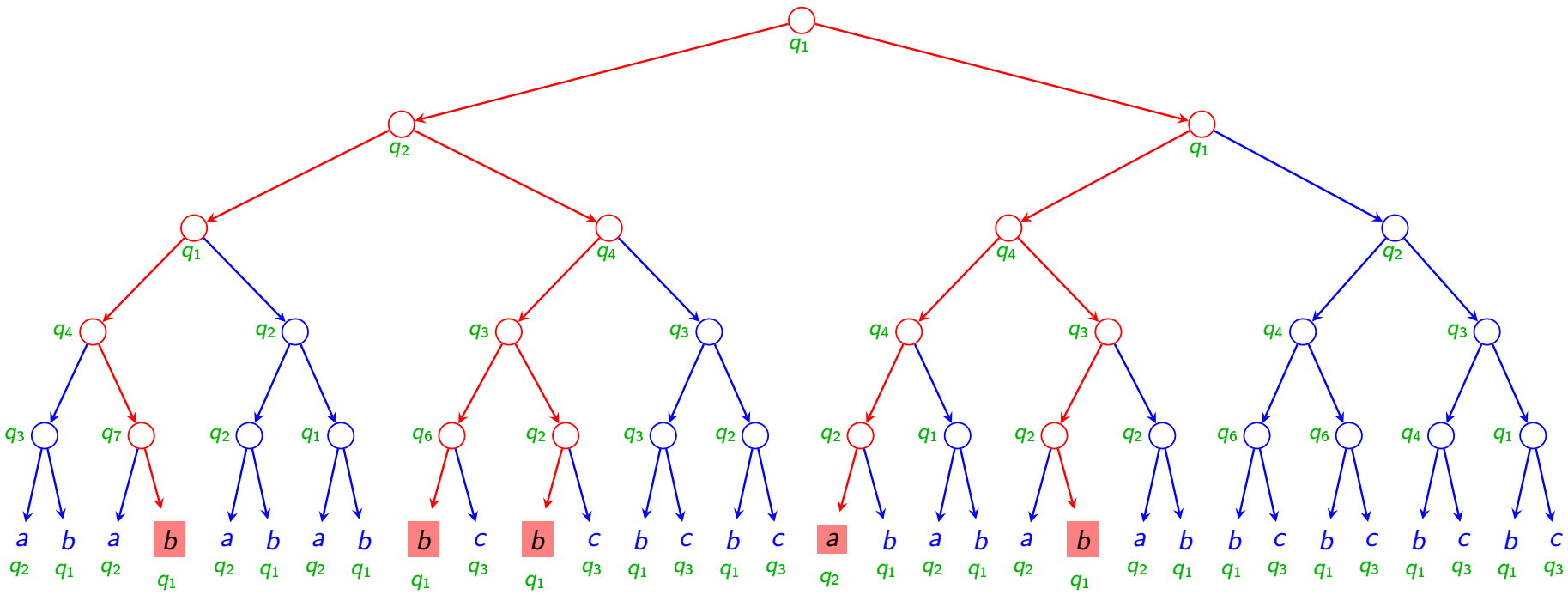

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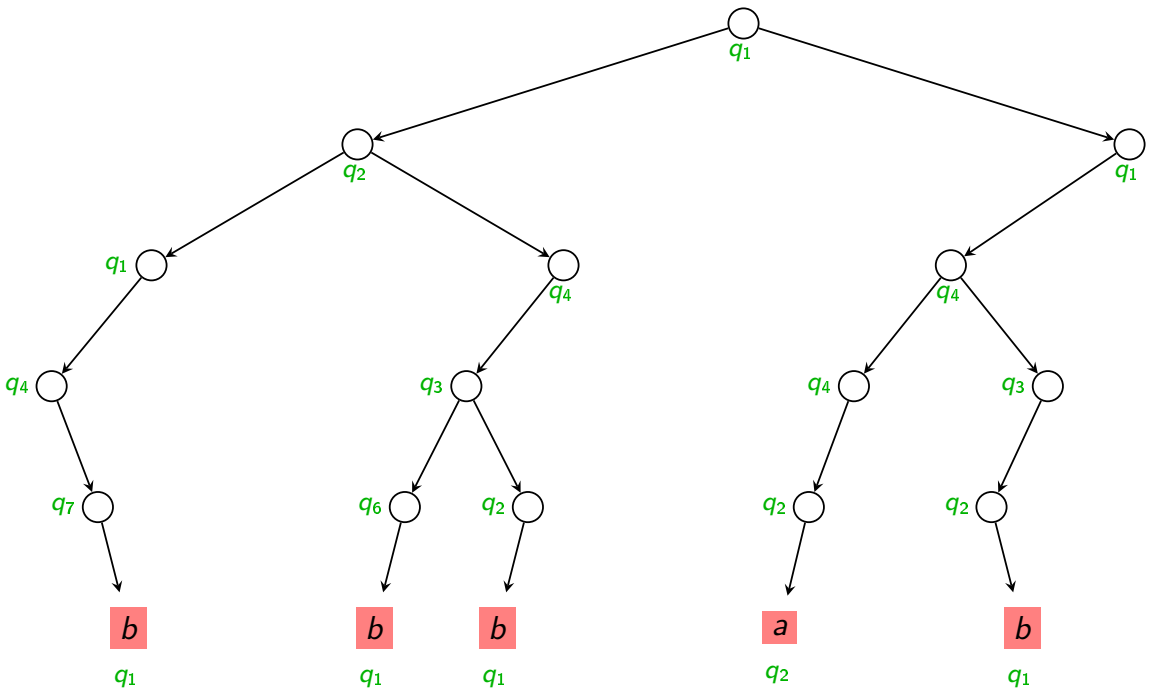

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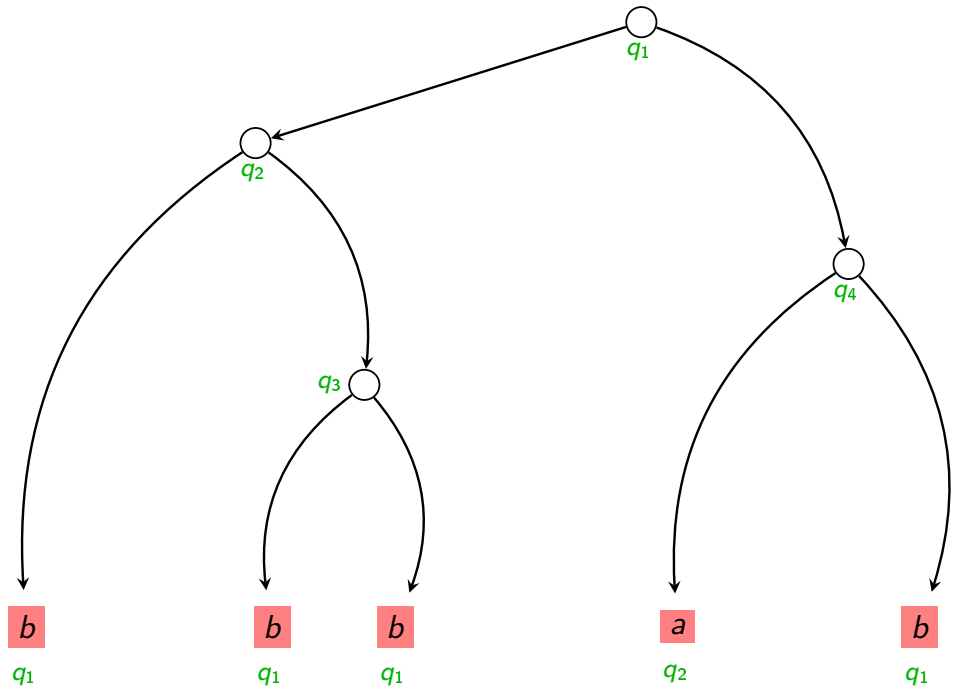

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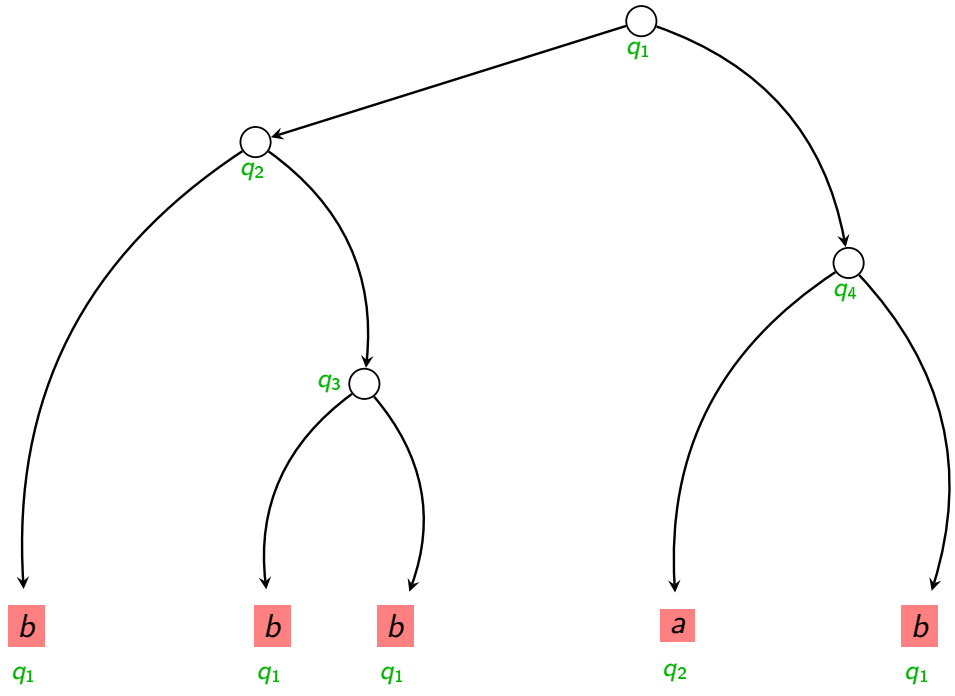
**Witness tree** for leaf-set $S$:



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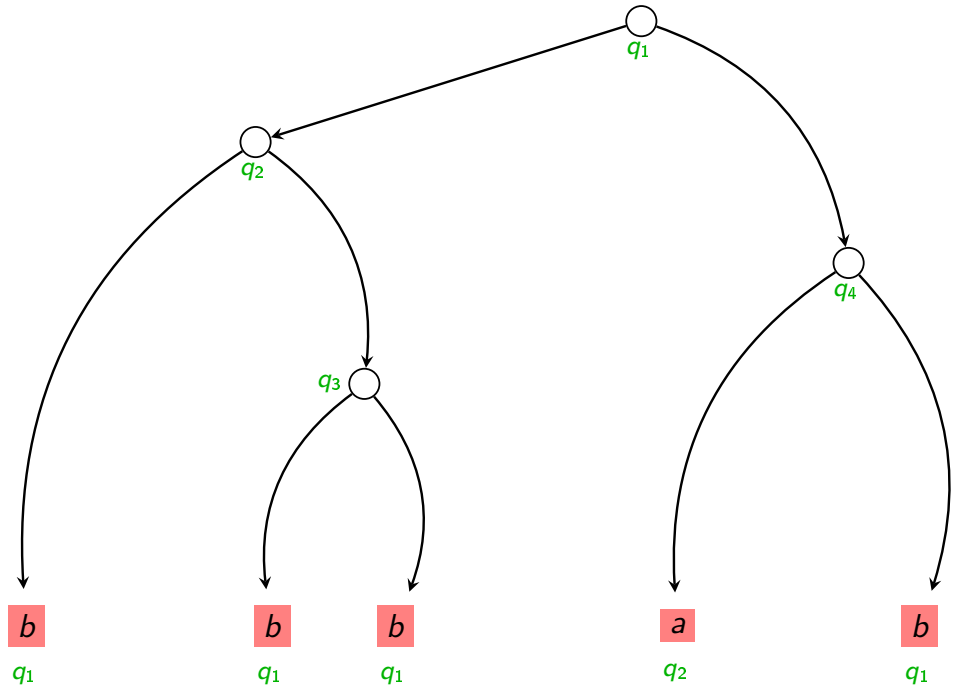


Every $(v, q) \in V \times Q$ is a **configuration**.

A configuration if **useful** if it is the node of some witness tree.

Arcs $(v, q) \to (u, p)$ of witness trees are called **shortcuts**, and $(u, p)$ is then a **shortcut successor** of $(v, q)$.

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The Shortcut forest allows us to compute all possible shortcut successors of a given $(v, q)$, which is enough to efficiently enumerate the witness trees.

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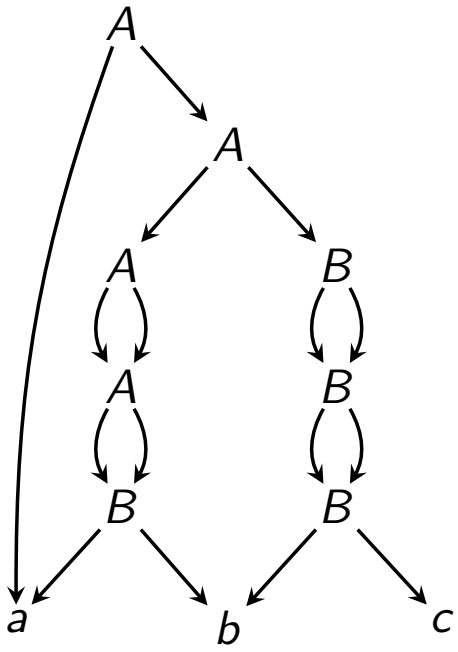
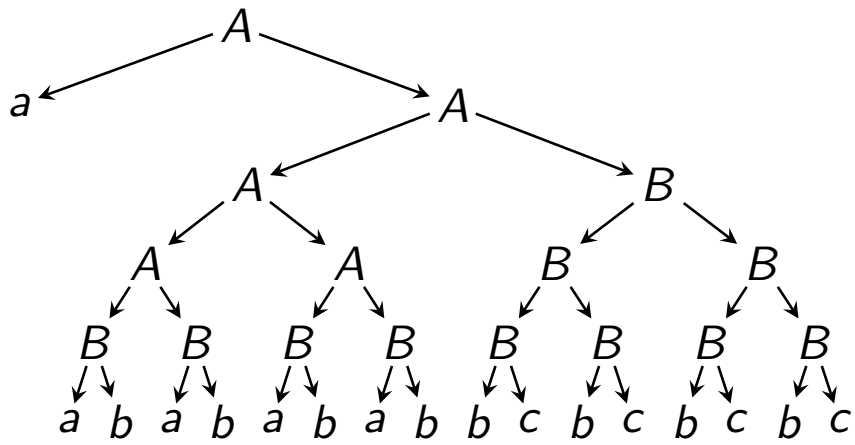
We want to carry out Bagan's algorithm, but the binary tree is given as its DAG-folding.




DAG-folding of the tree.

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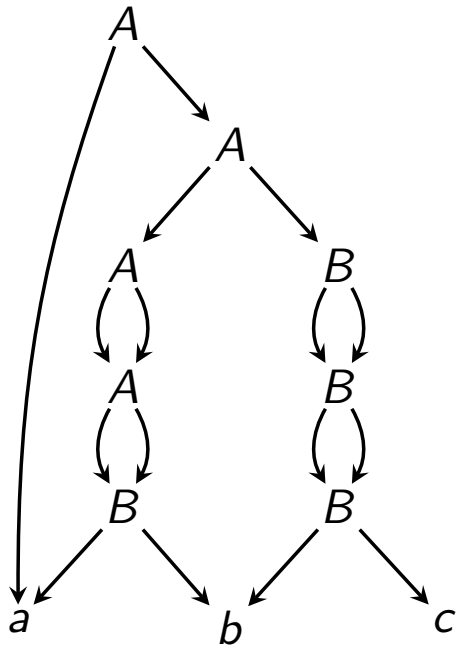


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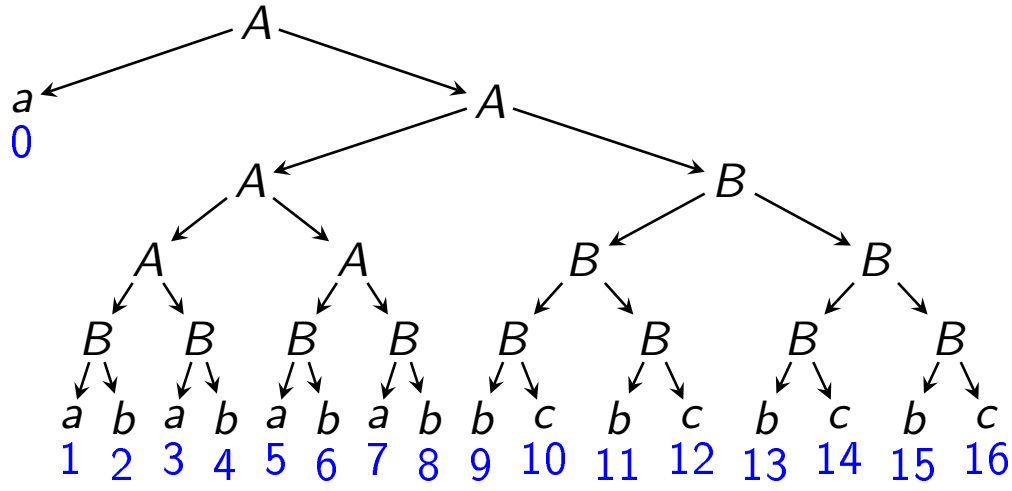


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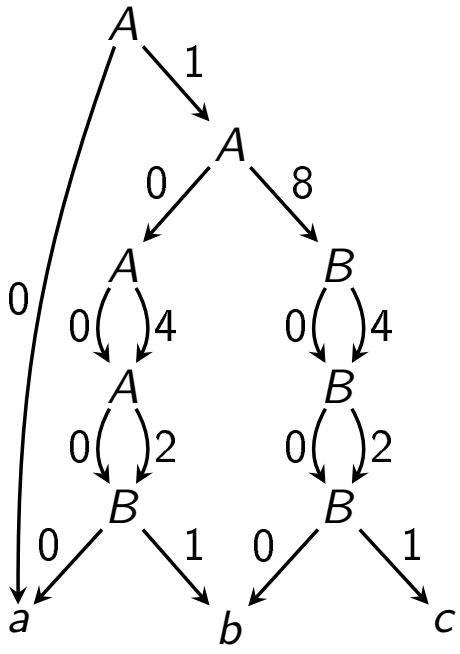


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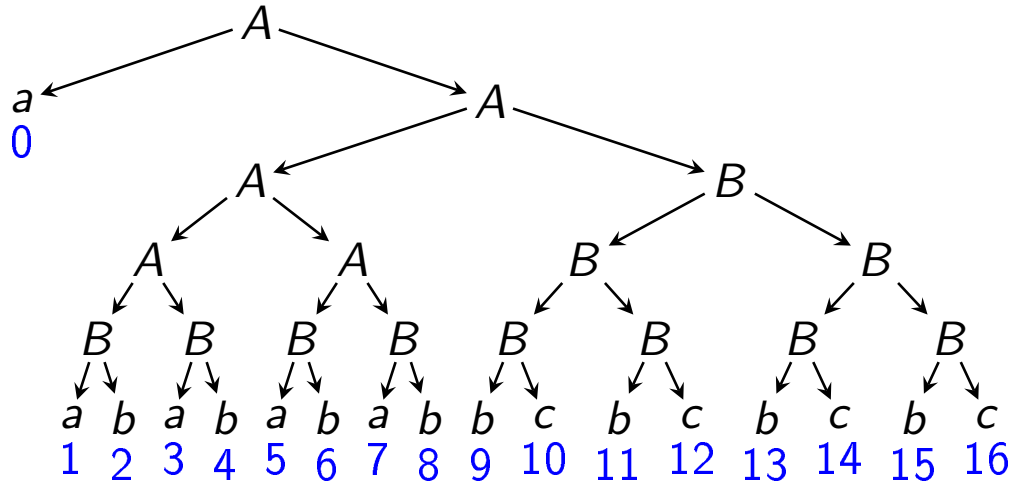


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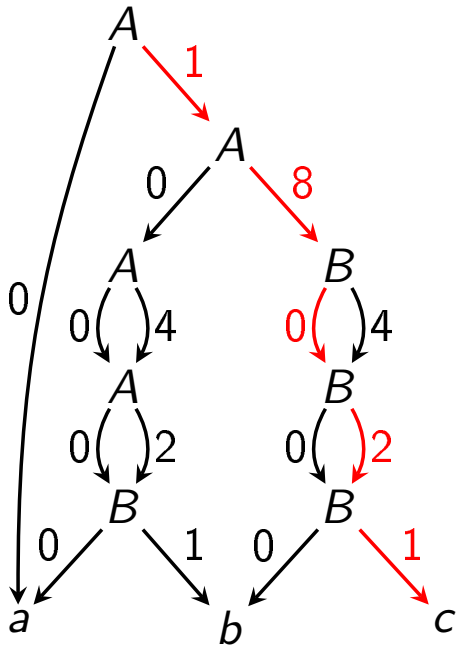


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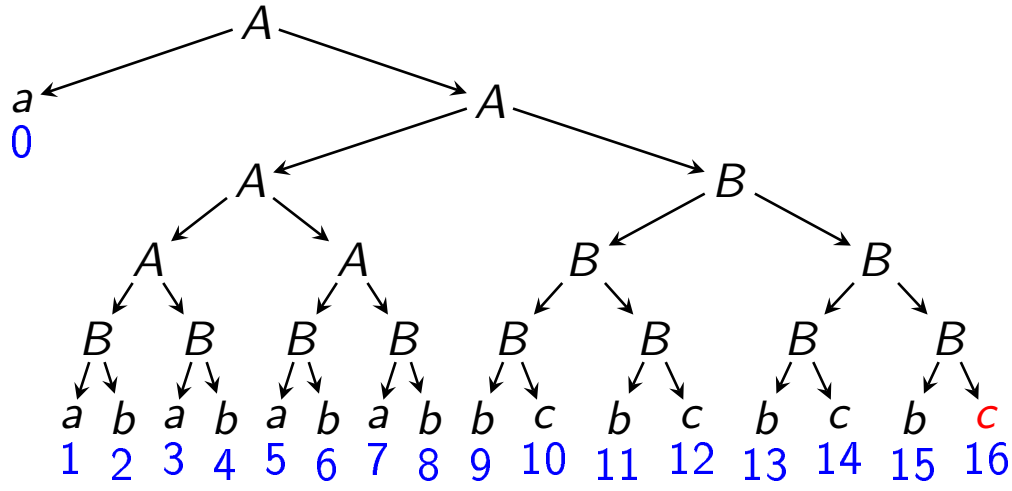


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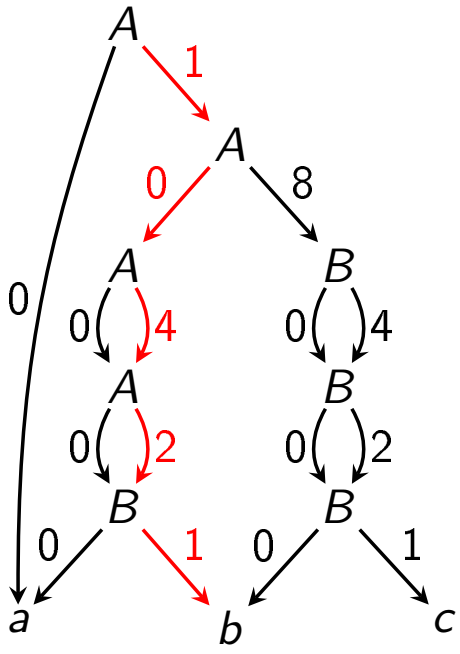


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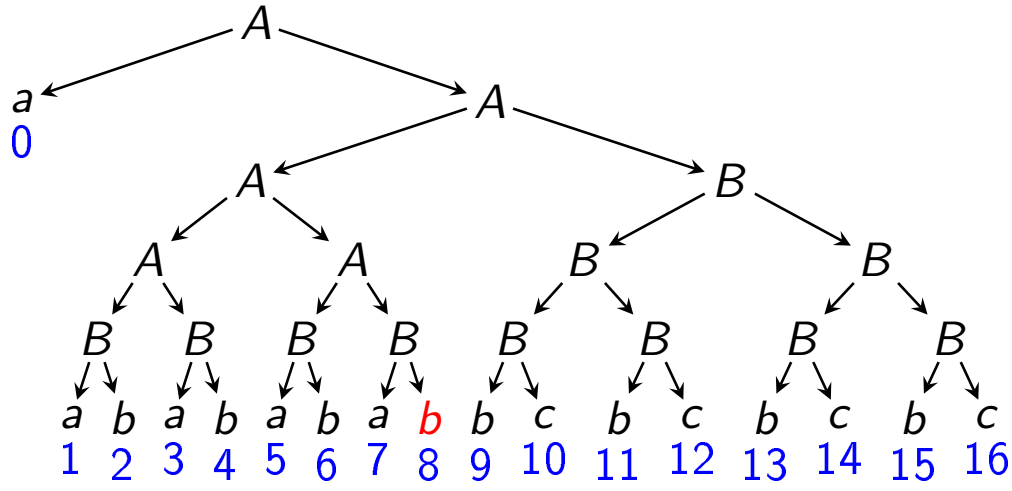


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Problem: We cannot afford to compute the full shortcut forest

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Solution: We can compute the DAG-folding of the shortcut forest.

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This boils down to the following path enumeration problem in DAGs.

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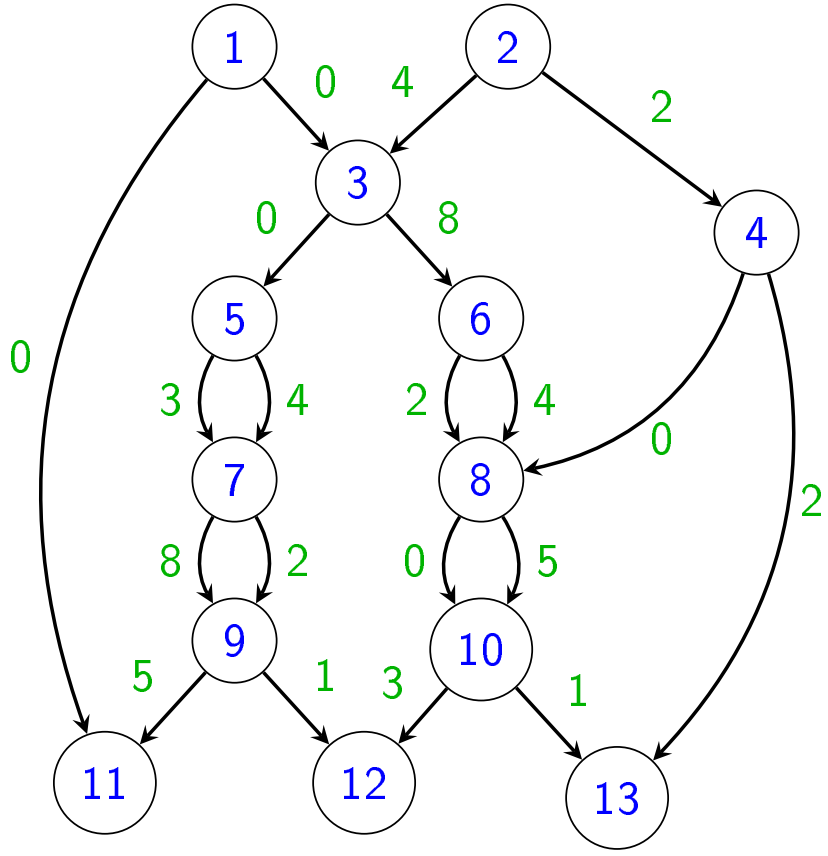
Let $D = (V, E)$ be a binary DAG with weight function $\gamma : E \to M$.



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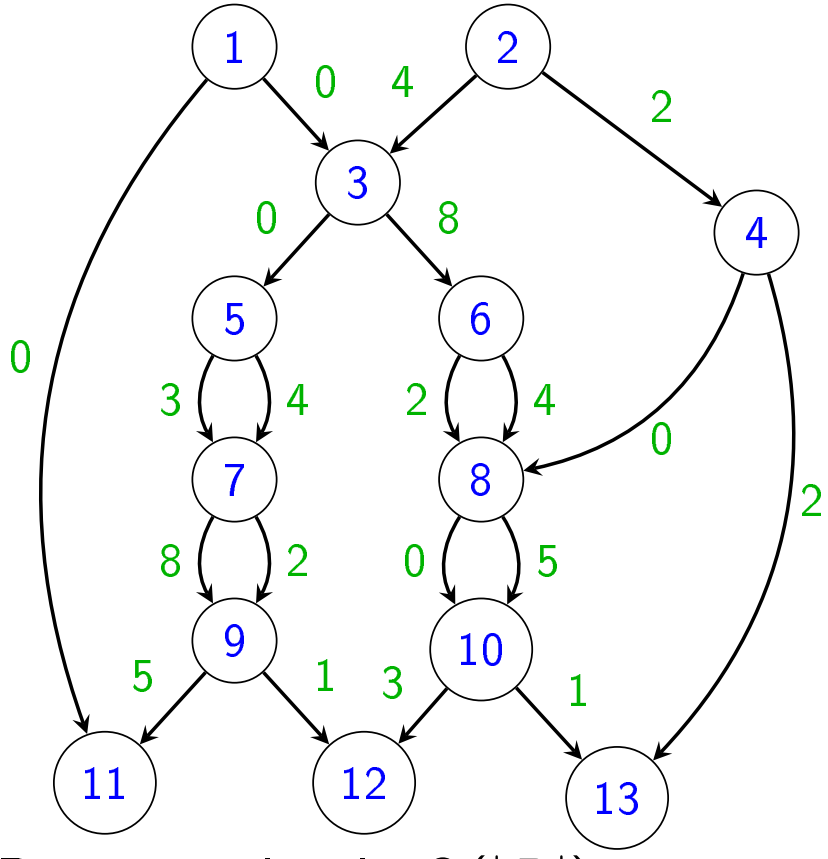


Preprocessing in $O(|D|)$.

For any $s \in V$, enumerate with constant delay all paths $\pi$ from $s$ to some leaf $u$ represented by $(u, \gamma(\pi))$.

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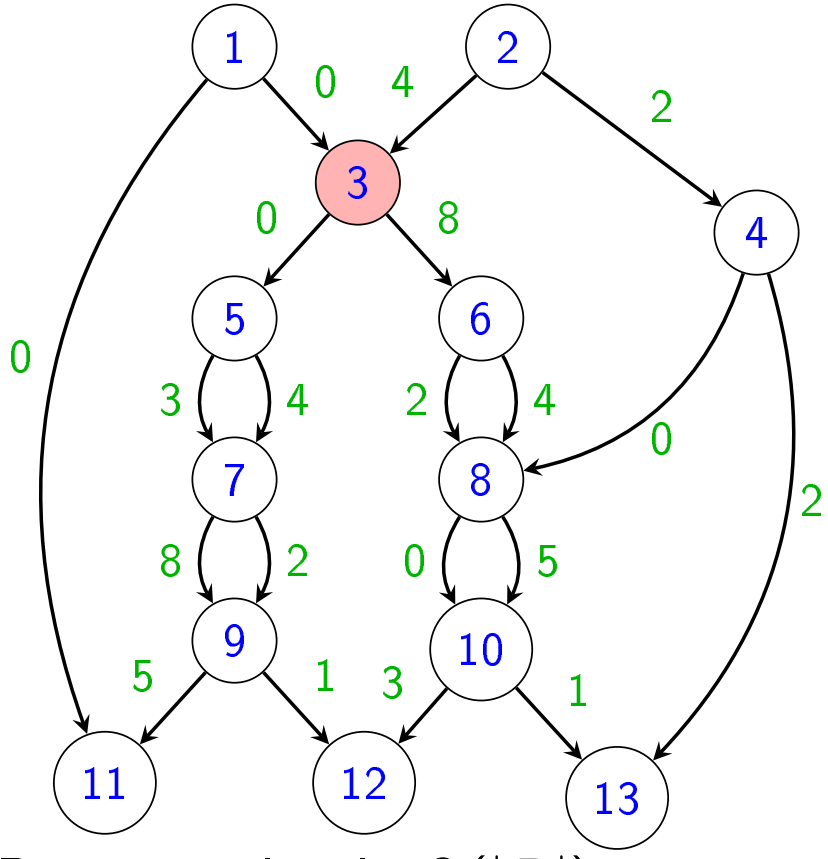


Enumeration for start node 3:

Preprocessing in $O(|D|)$.

For any $s \in V$, enumerate with constant delay all paths $\pi$ from $s$ to some leaf $u$ represented by $(u, \gamma(\pi))$.

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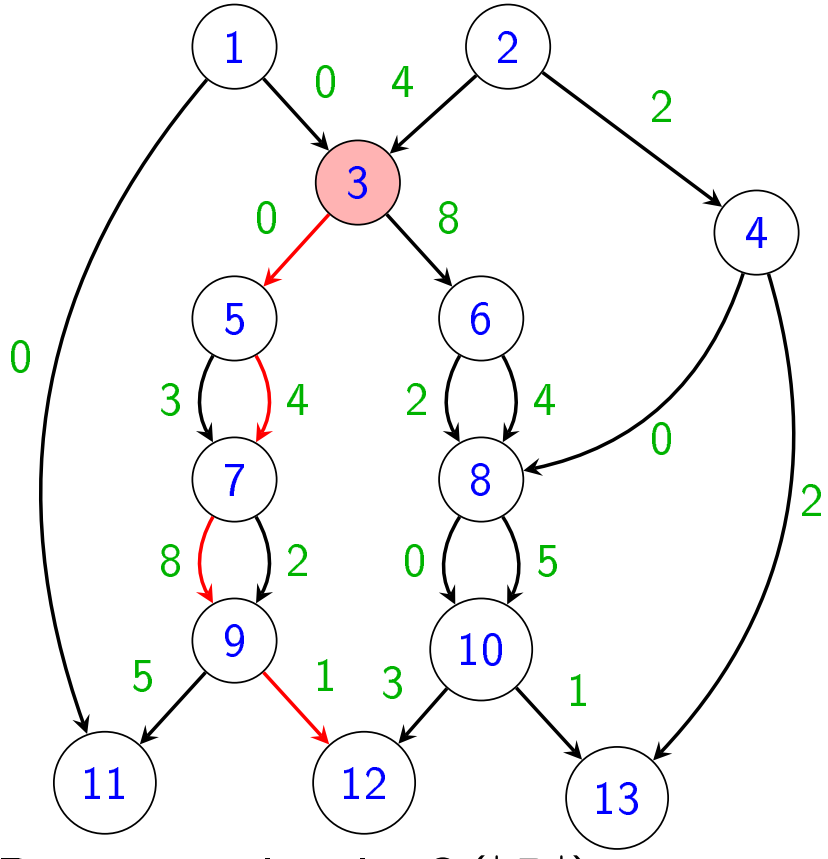
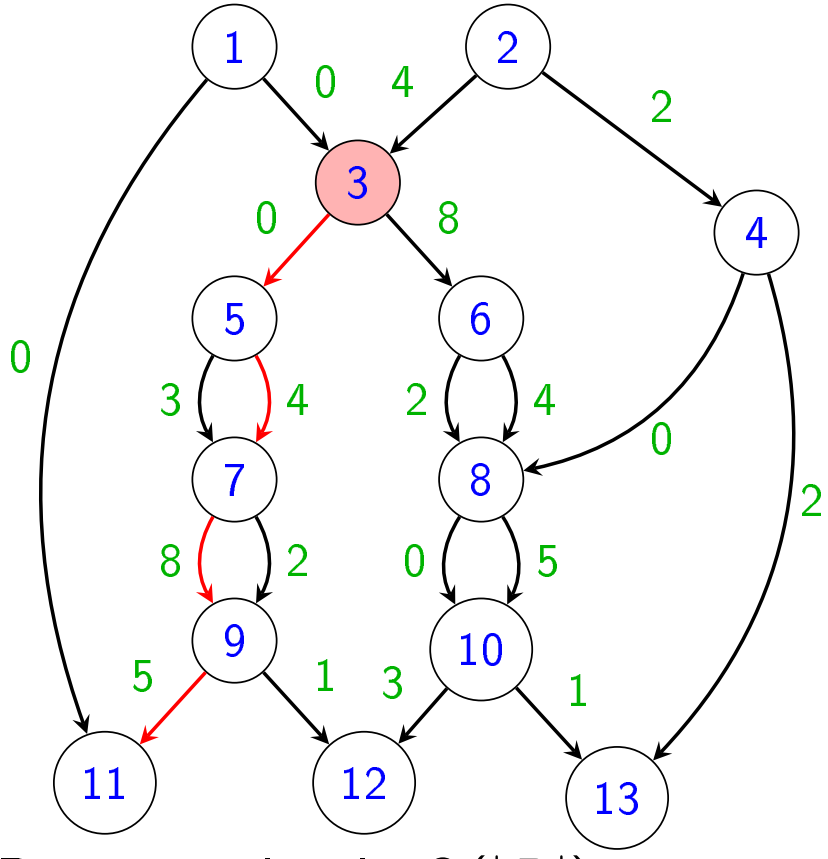



Enumeration for start node 3:
$(12, 13)$

Preprocessing in $O(|D|)$.
For any $s \in V$, enumerate with constant delay all paths $\pi$ from $s$ to some leaf $u$ represented by $(u, \gamma(\pi))$.

# Bagan's Algorithm for DAG-Foldings

Let $D = (V, E)$ be a binary DAG with weight function $\gamma : E \to M$.




Enumeration for start node 3:
$(12, 13), (11, 17)$

Preprocessing in $O(|D|)$.
For any $s \in V$, enumerate with constant delay all paths $\pi$ from $s$ to some leaf $u$ represented by $(u, \gamma(\pi))$.

# Bagan's Algorithm for DAG-Foldings

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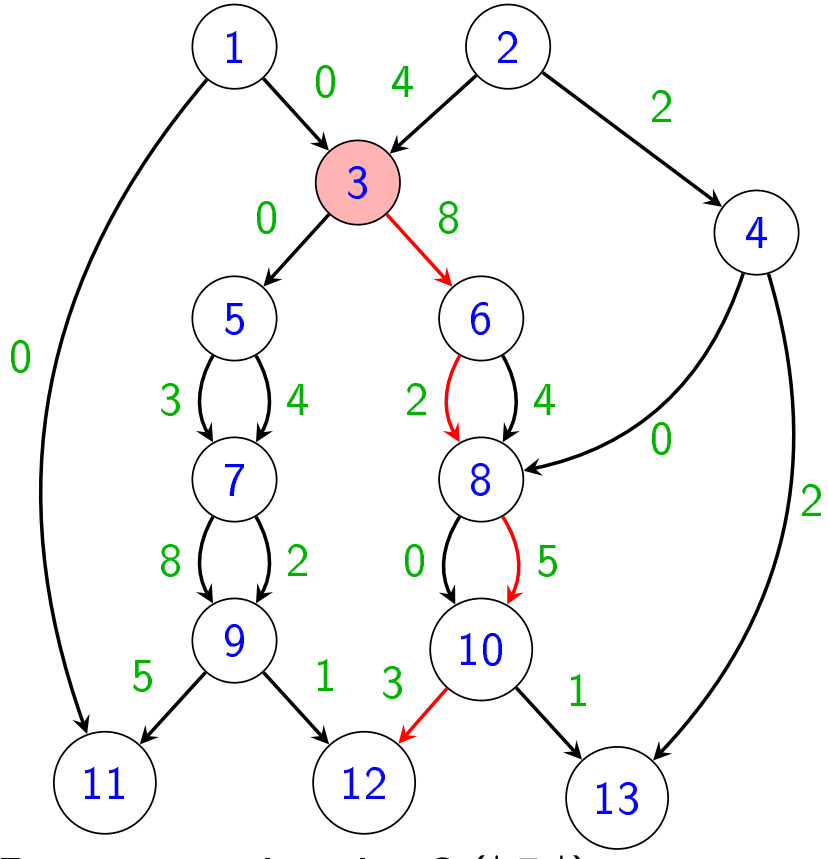


Enumeration for start node 3:
$(12, 13), (11, 17), (12, 18), \ldots$

Preprocessing in $O(|D|)$.
For any $s \in V$, enumerate with constant delay all paths $\pi$ from $s$ to some leaf $u$ represented by $(u, \gamma(\pi))$.

Thank you very much for your attention.