

Query Evaluation over SLP-Compressed Data

Markus Schmid
HU Berlin

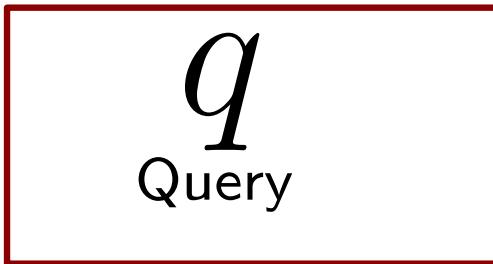
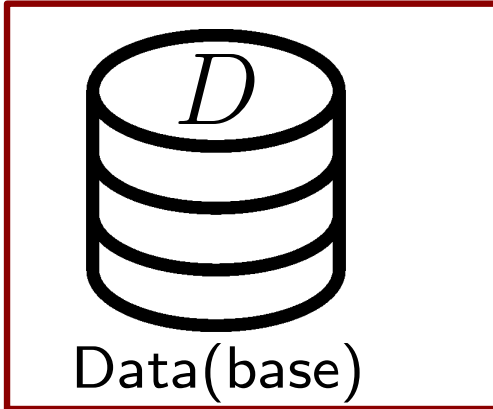
Forschungsvortrag
Magdeburg, 13.11.25

Outline of My Recent Research Programme

DFG Project: “Query Evaluation over SLP-Compressed Data”

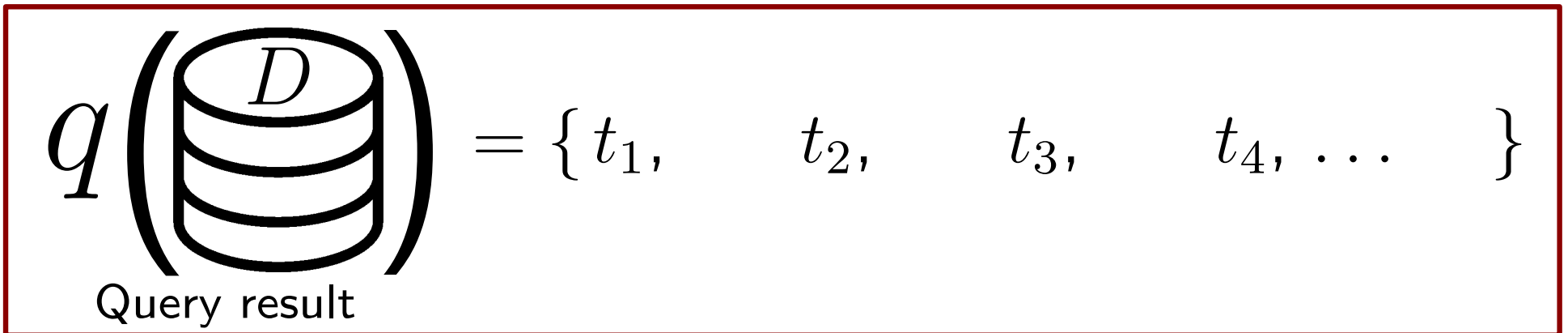
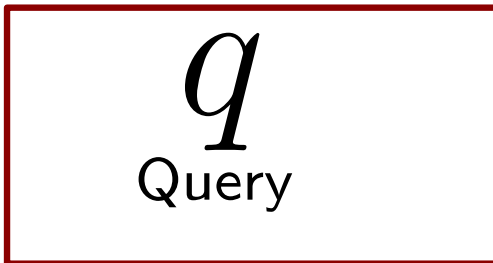
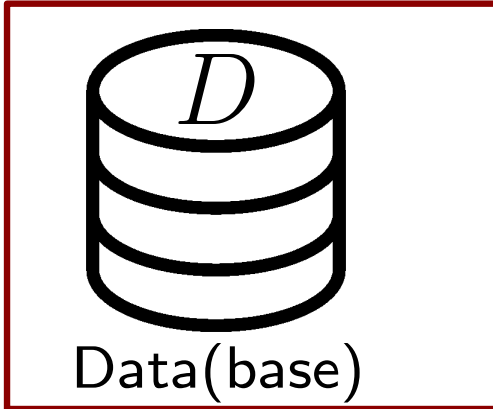
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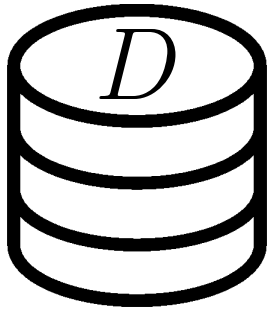
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Data(base)

Relational data

q
Query

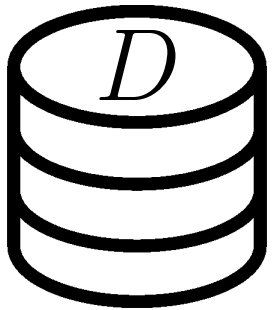
SQL-queries (FO-formulas)

$$q \left(\text{cylinder with } D \right) = \{ t_1, \quad t_2, \quad t_3, \quad t_4, \dots \}$$

Query result

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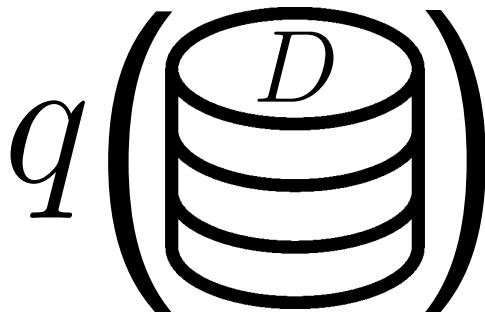


Data(base)

Relational data, strings,
trees, graphs, etc.

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SQL-queries (FO-formulas),
formulas in MSO,
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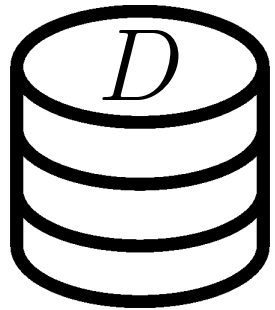


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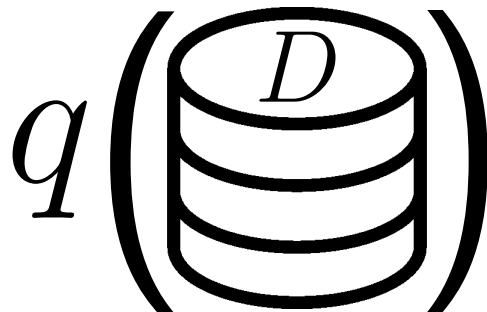


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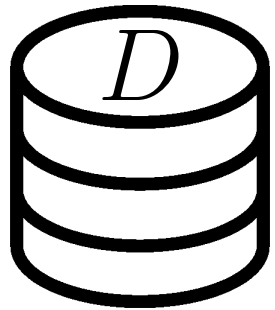
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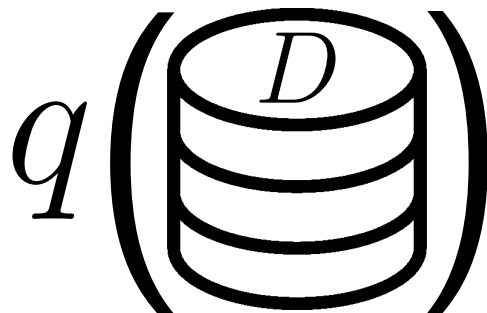


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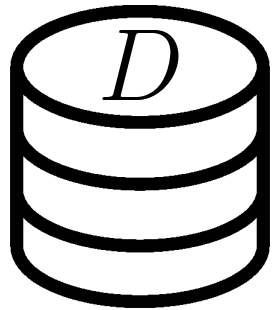
Preprocessing

$\{ t_1, \underset{\text{delay}}{|} t_2, \underset{\text{delay}}{|} t_3, \underset{\text{delay}}{|} t_4, \dots \}$

.....
enumerate 

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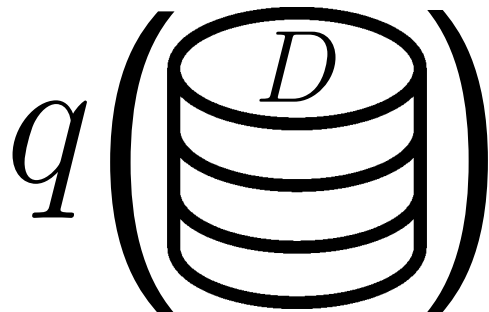


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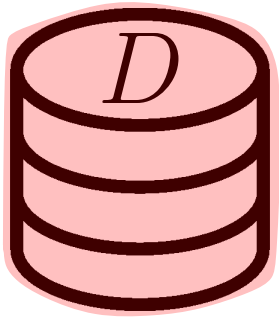


enumerate

Preprocessing: linear in $|D|$ Delay: Independent from $|D|$

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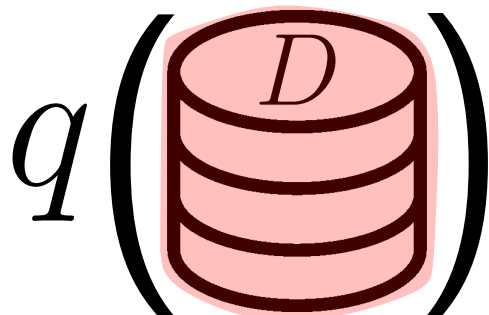


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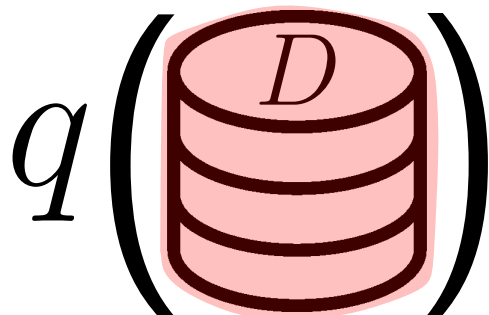


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Query

SQL-queries (FO-formulas),
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$$q\left(\text{Compressed data}\right) = \text{Preprocessing} \left\{ t_1, \underset{\substack{| \\ \text{delay} |}}{t_2}, \underset{\substack{| \\ \text{delay} |}}{t_3}, \underset{\substack{| \\ \text{delay} |}}{t_4}, \dots \right\}$$

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The Advantages of Querying Compressed Data

Manage data completely in compressed form!

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Algorithm B might outperform algorithm A on strongly compressible instances!

Querying Compressed Strings, Trees, and Relational Data

We will survey three results about query enumeration over SLP-compressed data.

Query class	Data domain
(1) Document spanners	Strings
(2) MSO-queries	Node-labelled unranked forests
(3) FO-queries	Relational structures of bounded degree

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Note: We measure algorithmic performance in data complexity. This means that the size of the query is constant.

The Compression Scheme

We need a compression scheme that ...

- ... is lossless.
- ... is versatile, i.e., we can use it for different data domains.
- ... potentially achieves very strong compression.
- ... is suitable for algorithms on compressed inputs.
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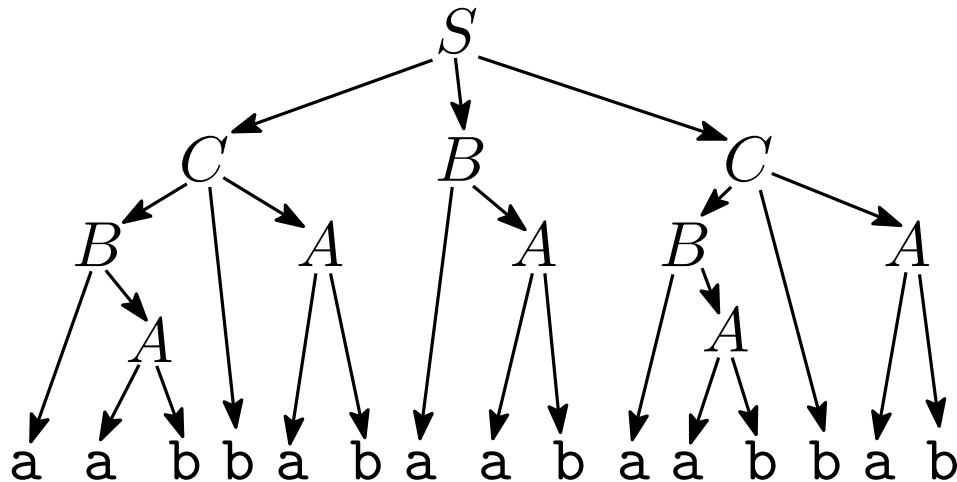
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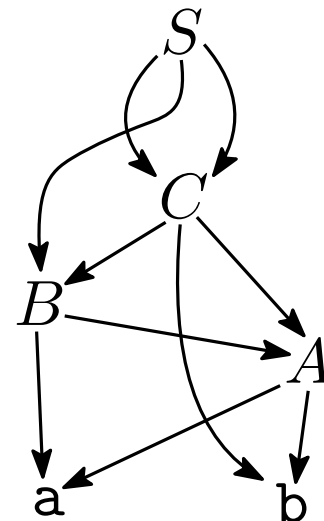
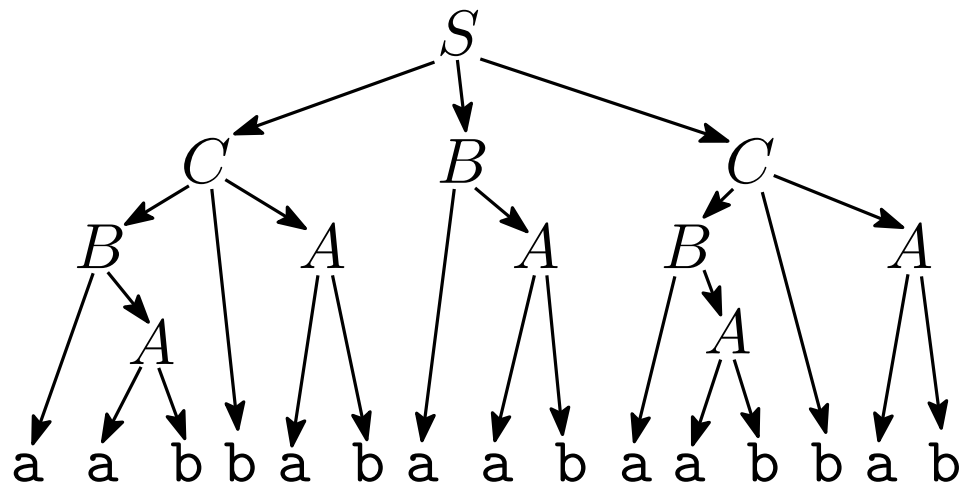
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String-SLPs - State of the Art

String-SLPs are very popular and have several good properties.

- exponential compression in the best case.
- tightly related to dictionary based compression, in particular LZ77 and LZ78.
- there are many fast compression algorithms that achieve excellent compression in practice.
- there are many fast algorithms that work directly on SLP-compressed strings.

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New angle and challenge: Extend known techniques ...

- ...to the query evaluation perspective.
- ...to the enumeration perspective.

Document Spanners Over SLP-Compressed Texts

Document spanners:

Interpret regular expressions (or automata) as queries

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$$q(\text{aaacbcacccbca}) =$$

x	y
(1, 3)	(5, 5)
(2, 3)	(5, 5)
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Theorem:

Given a document spanner q ,
a string D ,
after preprocessing $O(|D|)$, we can
enumerate $q(D)$ with constant delay.

[Bagan 2006]

[Amarilli et al. 2021]

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Given a document spanner q ,
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[Schmid, Schweikardt,
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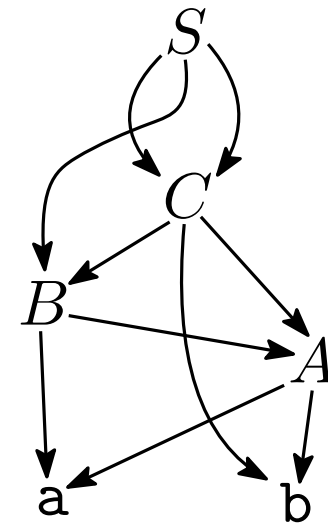
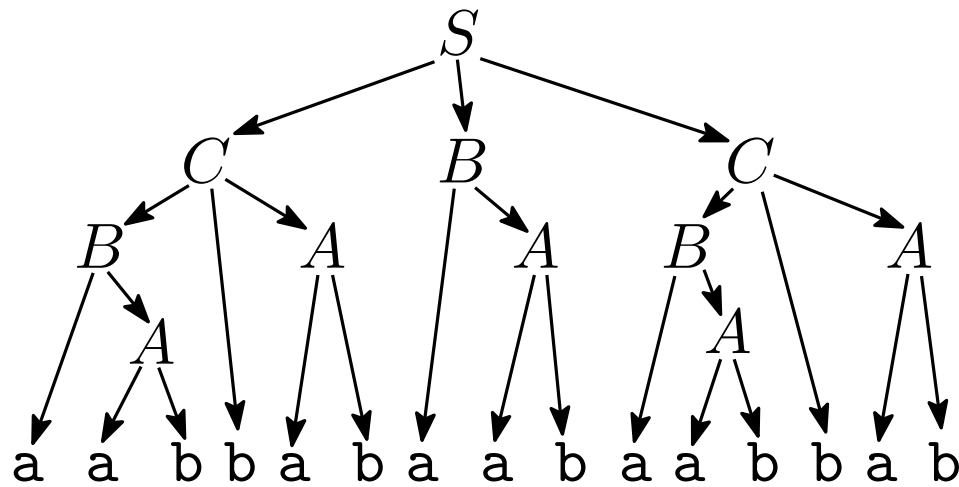
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Straight-Line Programs for Trees

Data Domain: Trees and Forests

Straight-Line Programs for Trees

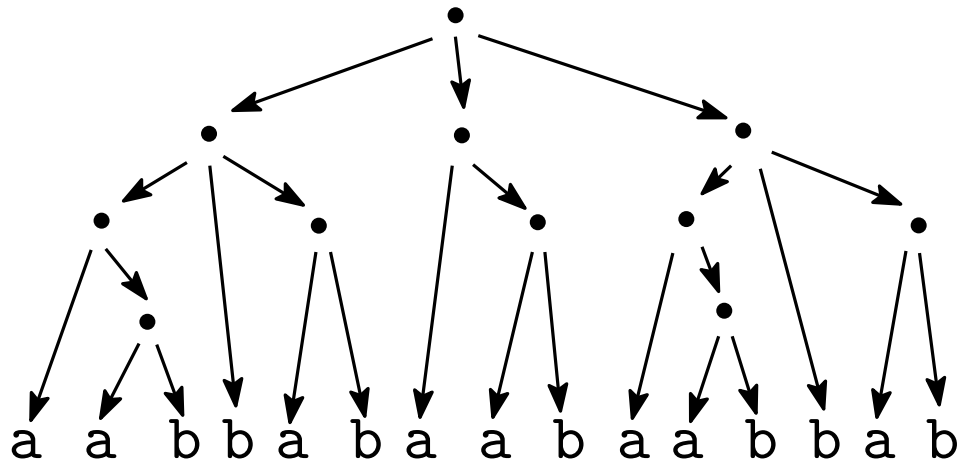
Recall string-SLPs: $S \rightarrow C B C, \quad B \rightarrow a A$
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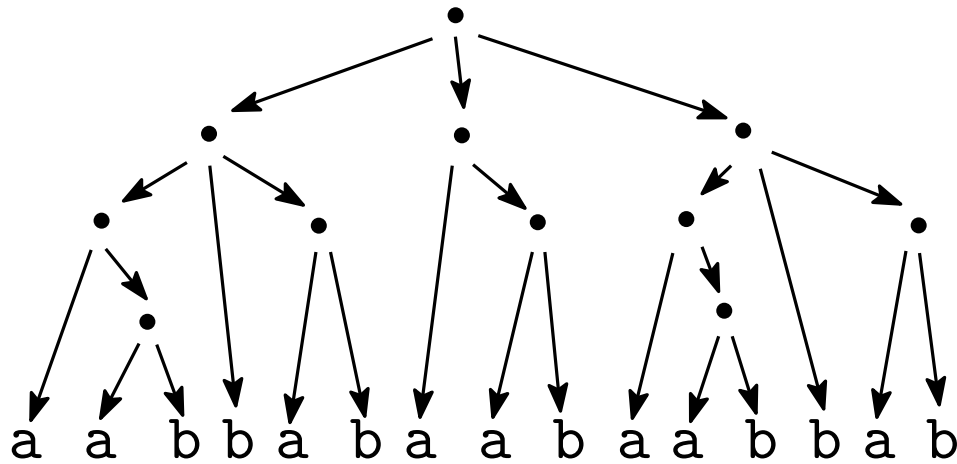
$$\begin{array}{ll} S \rightarrow C B C, & B \rightarrow \mathbf{a} A \\ C \rightarrow B \mathbf{b} A, & A \rightarrow \mathbf{a} \mathbf{b} \end{array}$$



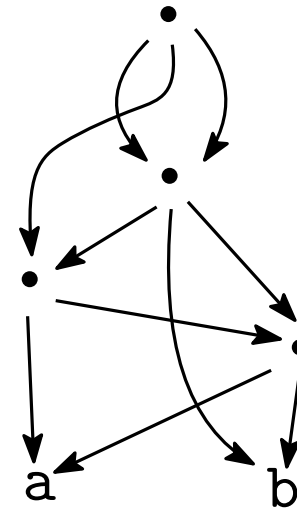
Expression over the monoid (Σ^*, \cdot)

Straight-Line Programs for Trees

Recall string-SLPs:

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Circuit over the monoid (Σ^*, \cdot)

Straight-Line Programs for Trees

[Gascón et al., ToCS 2020]

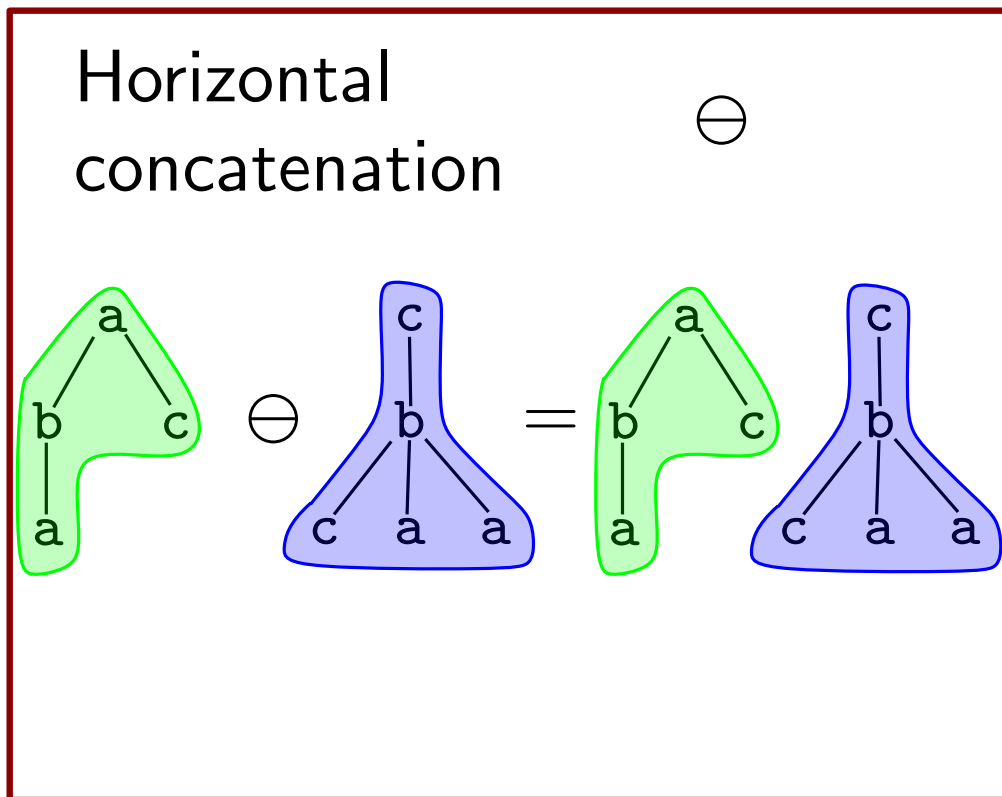
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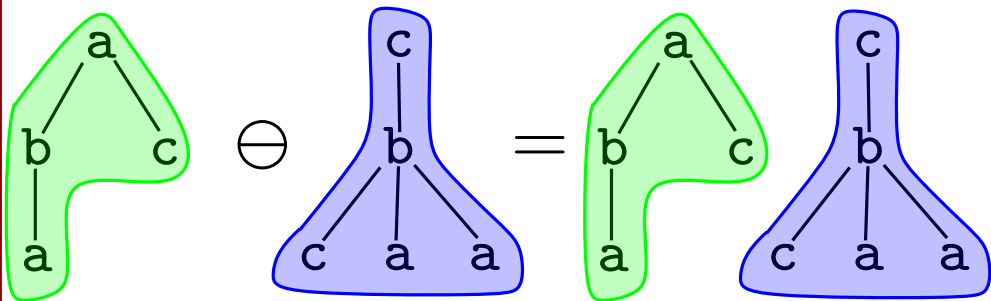
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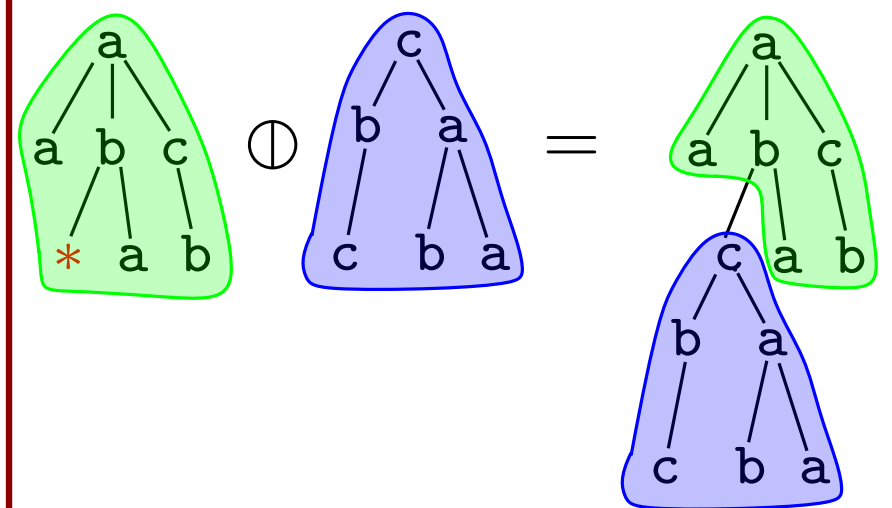
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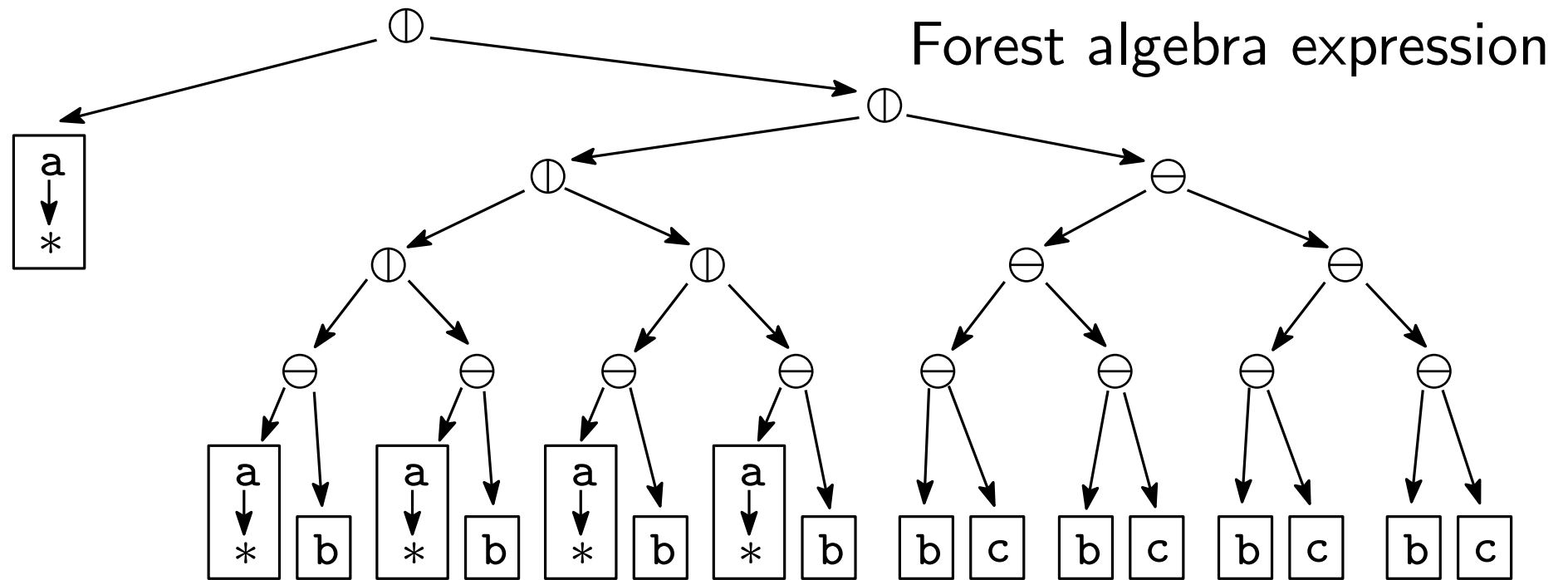


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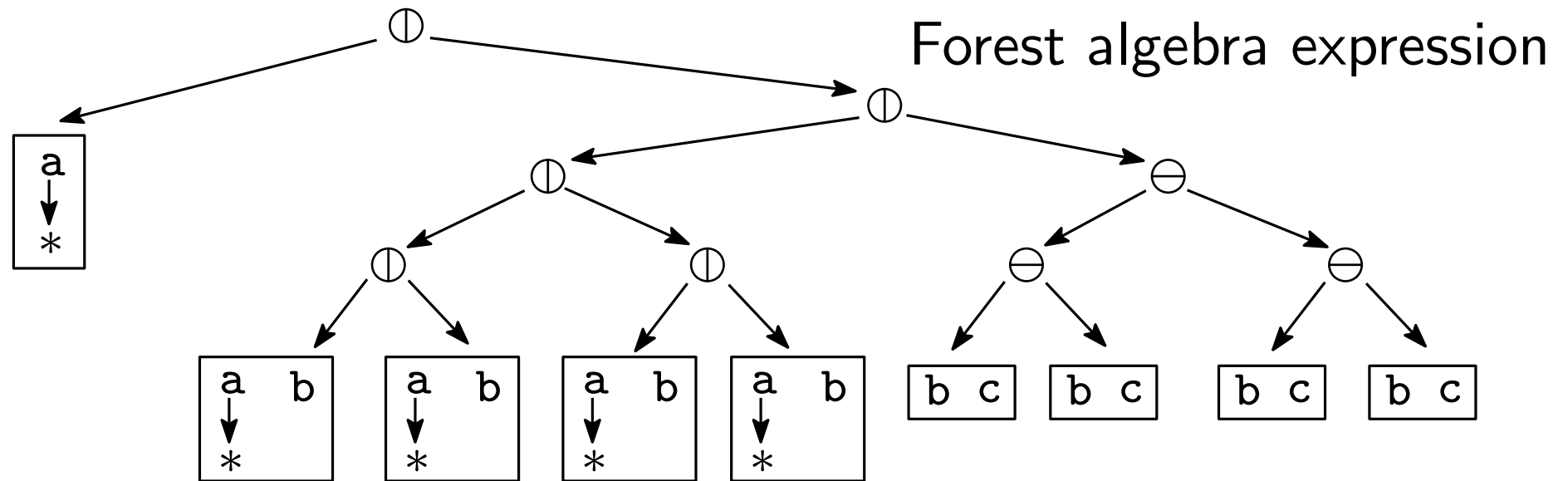
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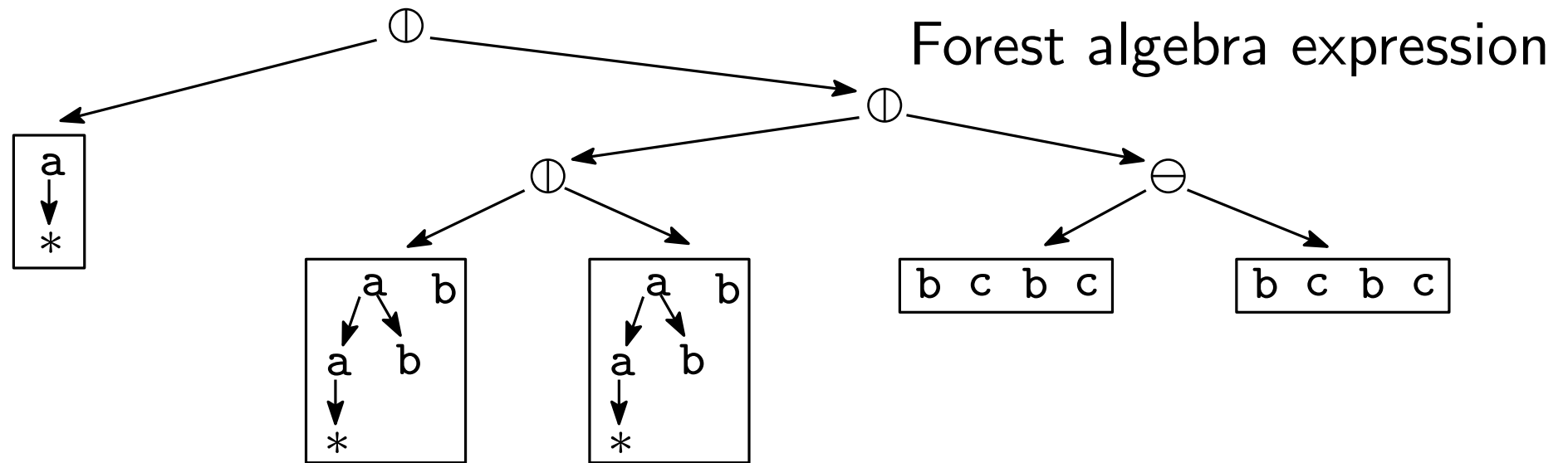
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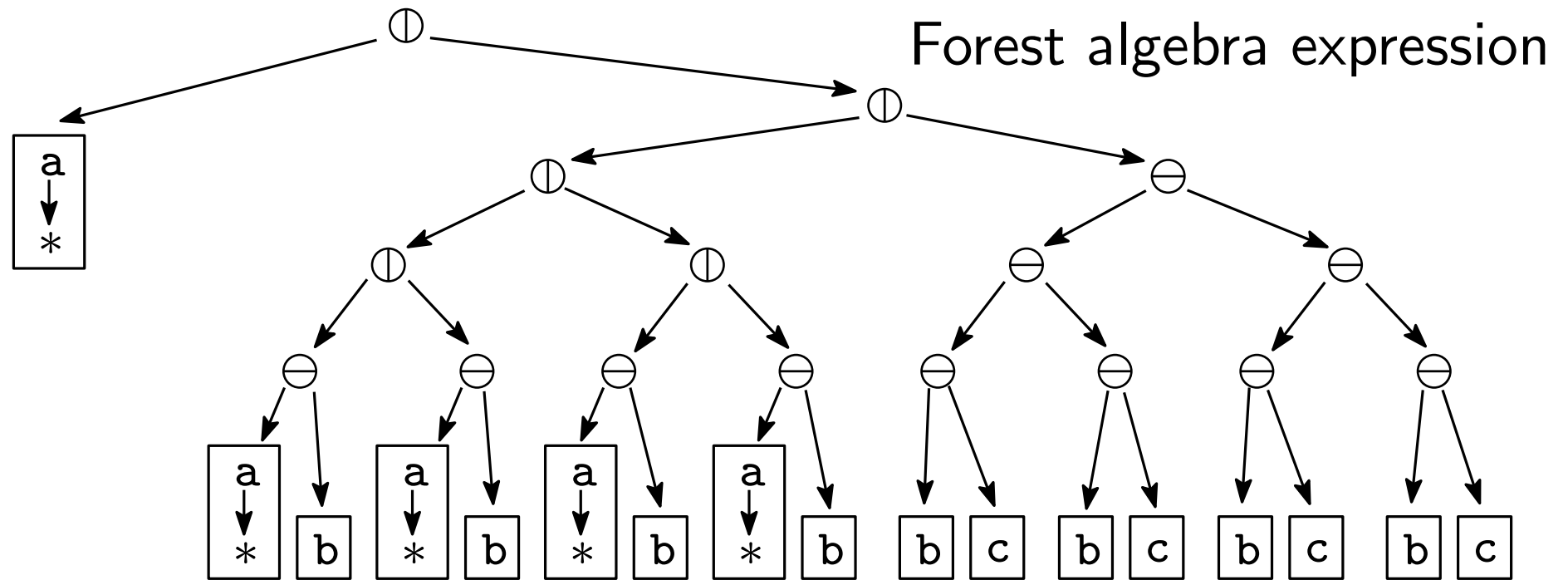
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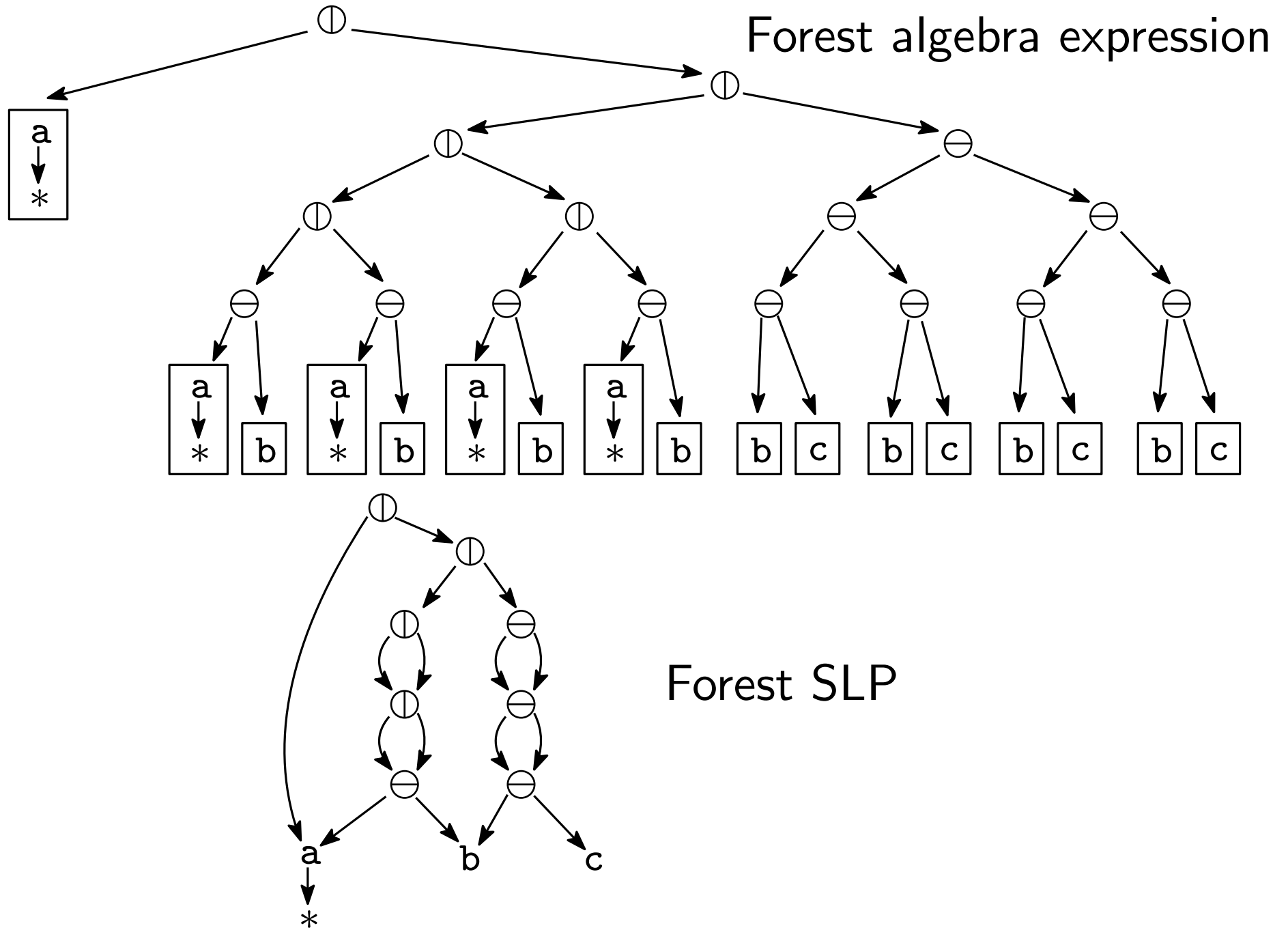
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MSO Enumeration Over SLP-Compressed Forests

Queries in monadic second order logic (MSO-queries)

Like first order logic, but also with quantifications over second order variables (i.e., set variables).

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Example: Query all (x, Z) from a tree such that x is a red node, Z is the set of x 's children and all nodes of Z are blue.

$$q(x, Z) = \text{red}(x) \wedge (\forall y \in Z : \text{blue}(y)) \wedge (\forall y : y \in Z \iff \text{child}(x, y))$$

MSO Enumeration Over SLP-Compressed Forests

Theorem:

Given an MSO query $q(X_1, \dots, X_m)$,
a node-labelled unranked forest F ,
after preprocessing $O(|F|)$, we can
enumerate $q(F)$ with output linear delay.

[Bagan, CSL 2006]

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Straight-Line Programs for Relational Structures

Data Domain: Relational Data

Straight-Line Programs for Relational Structures

Relational structure: $(\mathcal{U}, R_1, R_2, \dots, R_m)$

finite universe relations over \mathcal{U}

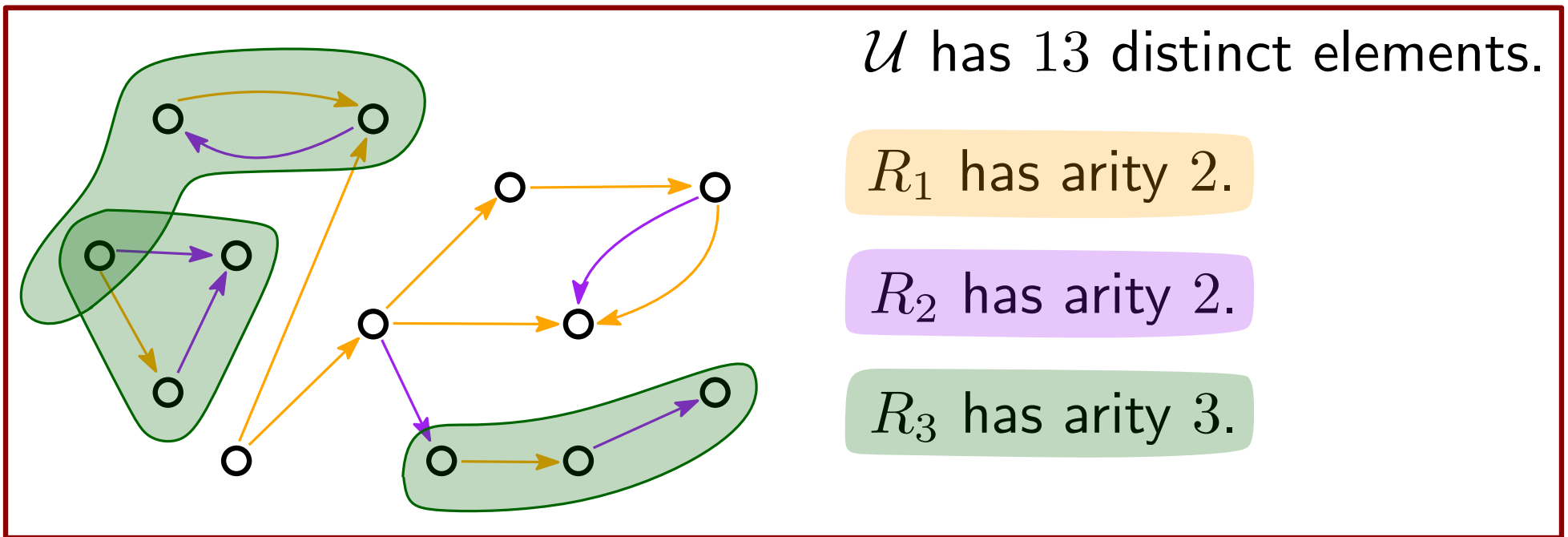
The diagram consists of two labels at the bottom: 'finite universe' on the left and 'relations over \mathcal{U} ' on the right. From 'finite universe', a straight arrow points diagonally up and to the right to the symbol \mathcal{U} in the tuple above. From 'relations over \mathcal{U} ', three curved arrows point diagonally up and to the left to the symbols R_1 , R_2 , and R_m in the tuple above.

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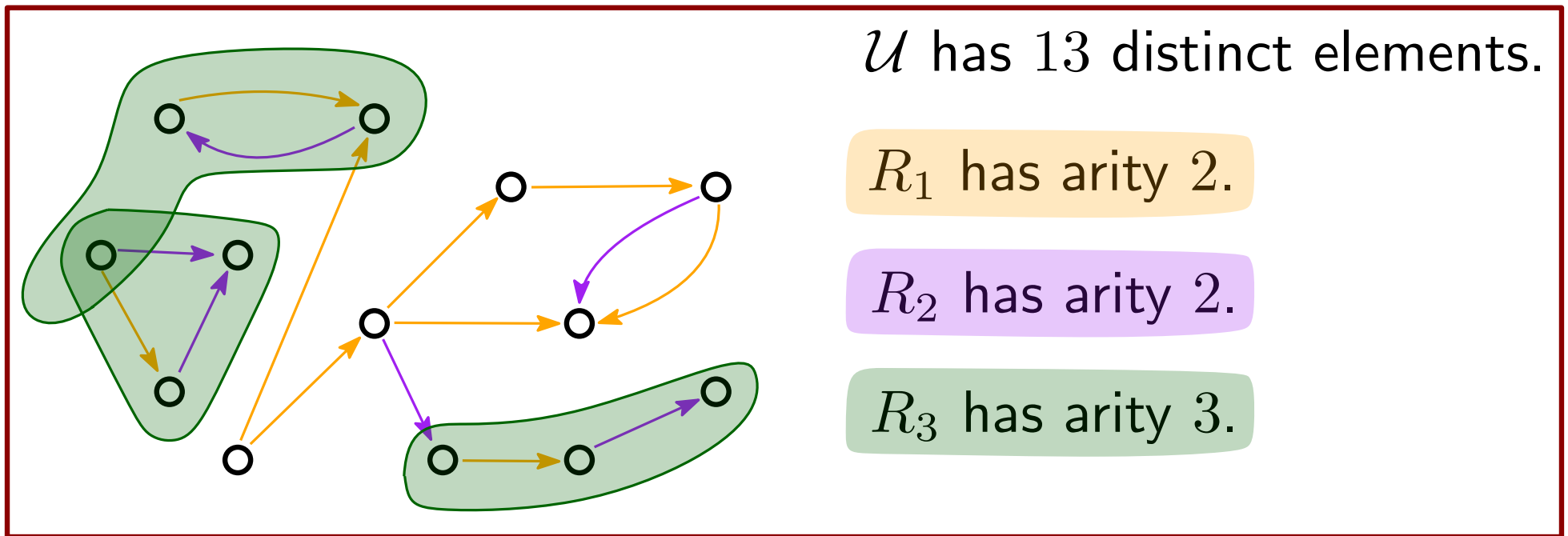


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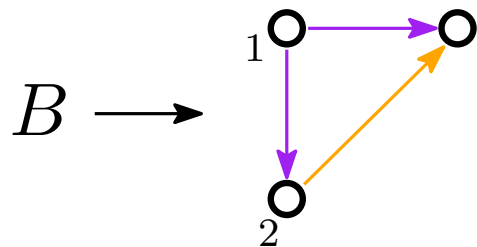
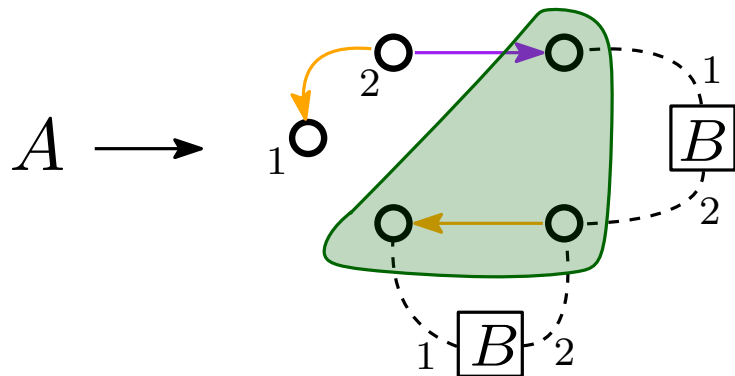
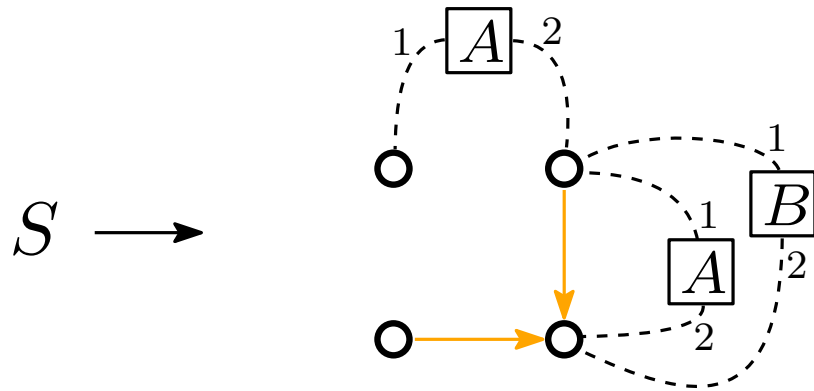


Relational straight-line programs: SLPs based on hyper-edge replacement grammars.

[Lengauer, Wanke, SICOMP 1988]

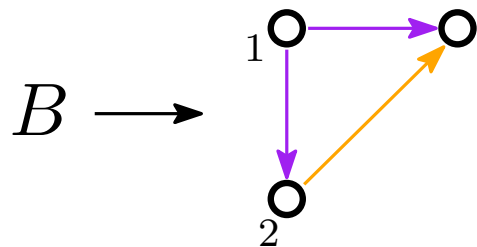
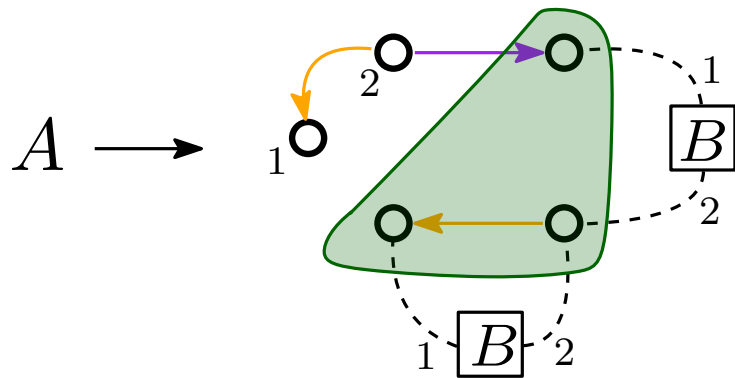
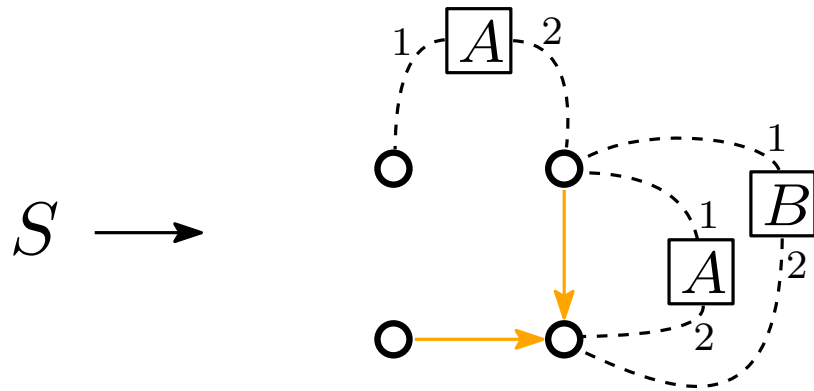
Straight-Line Programs for Relational Structures

Relational-SLP with 3 rules:

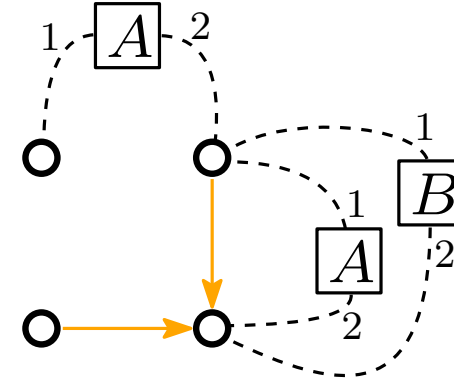


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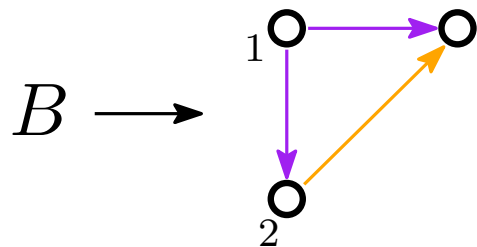
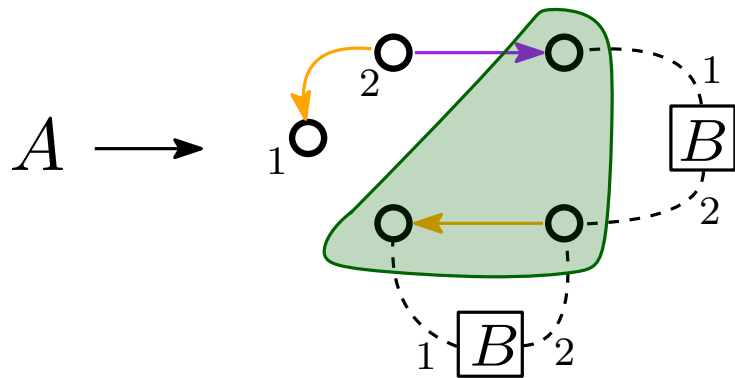
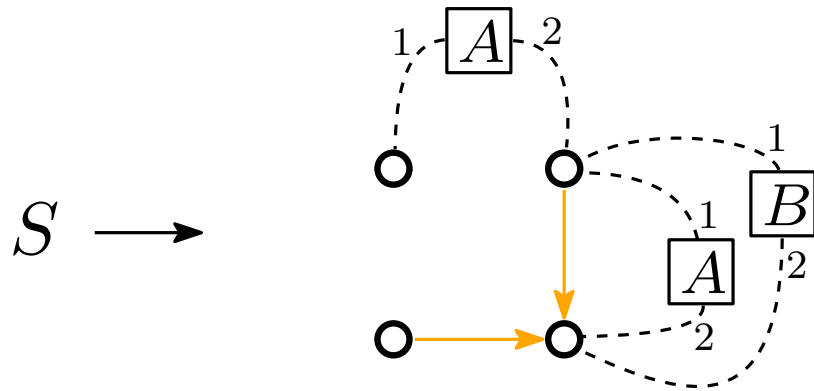


Derivation:

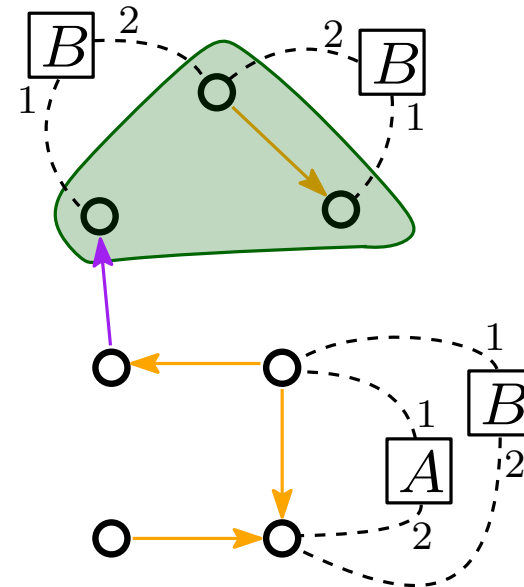


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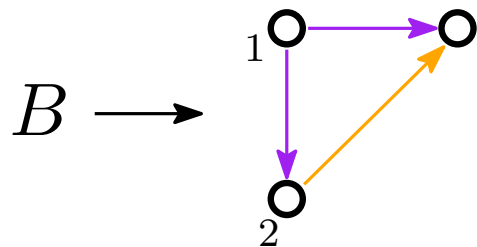
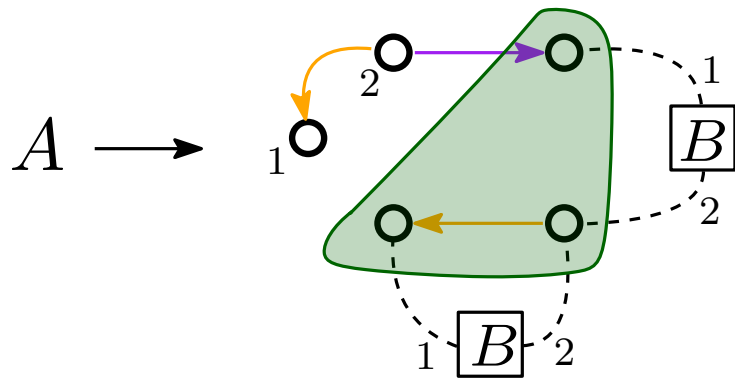
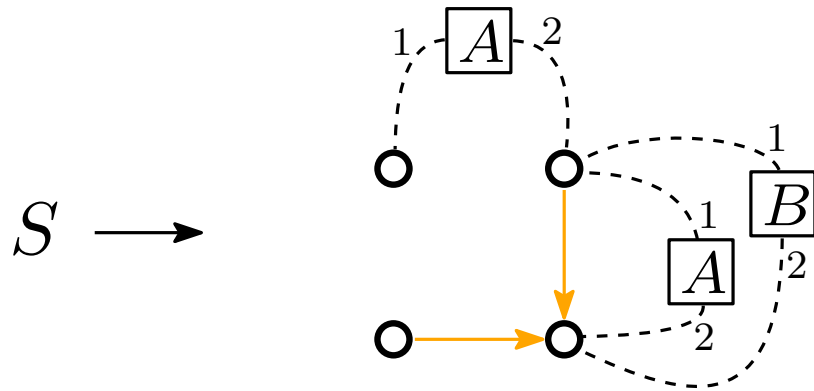


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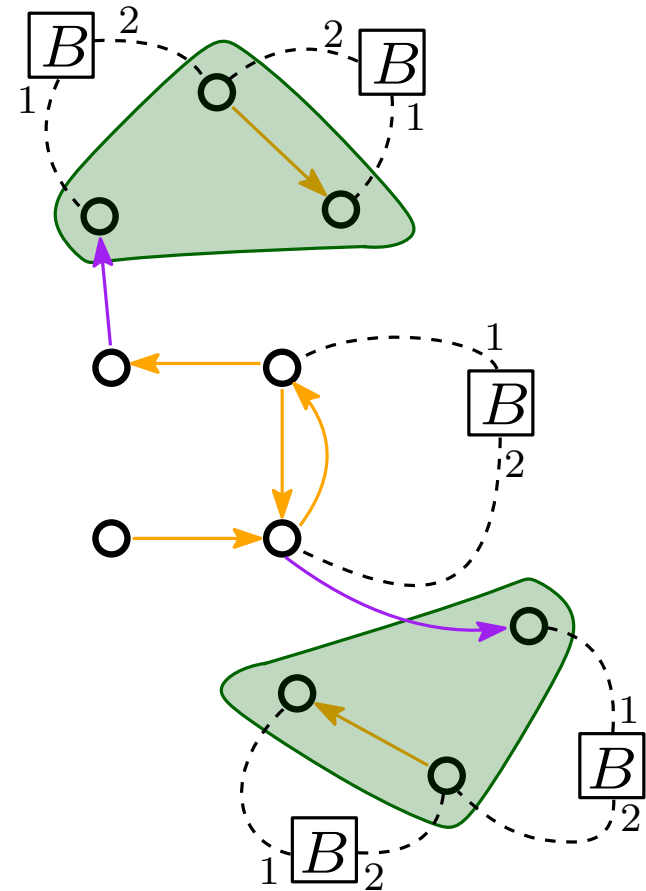


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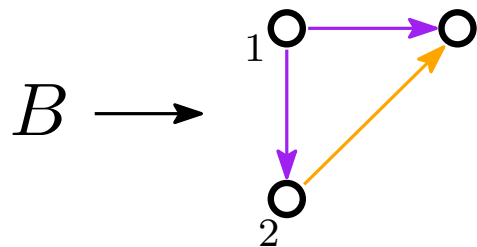
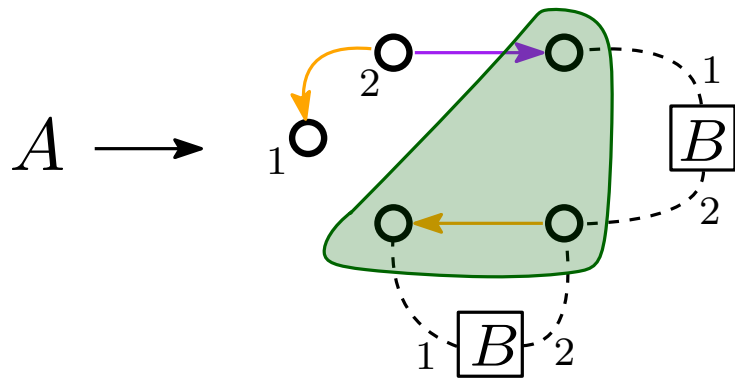
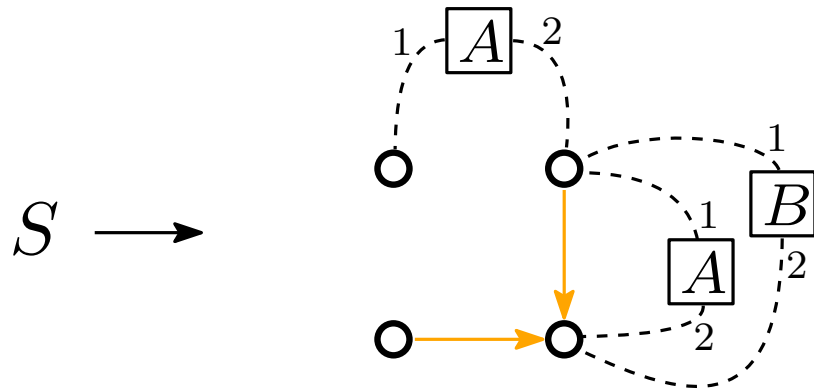


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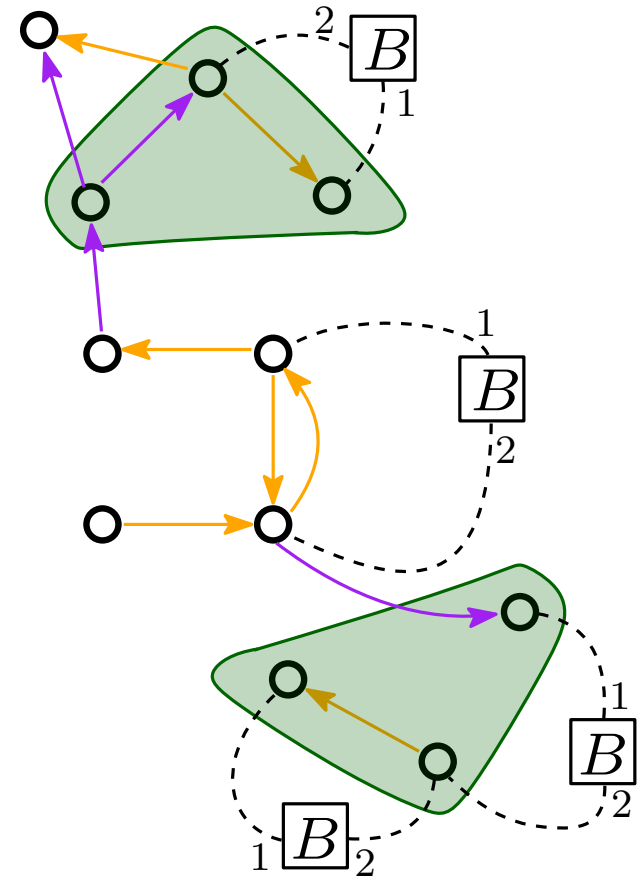


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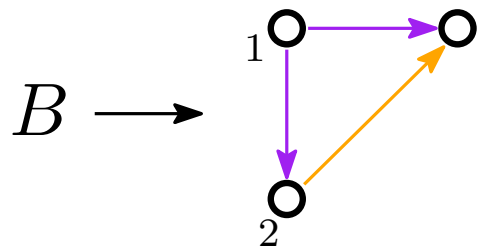
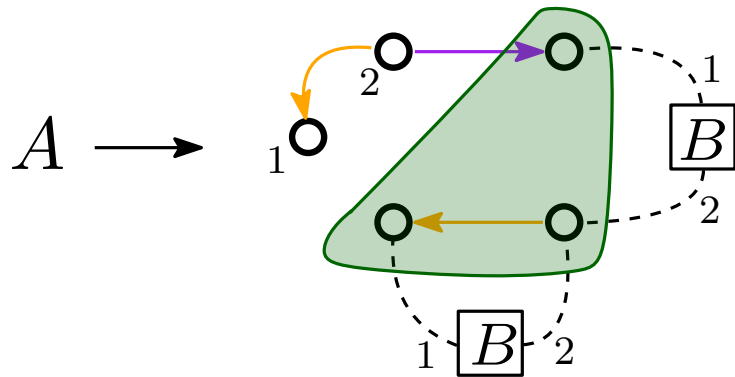
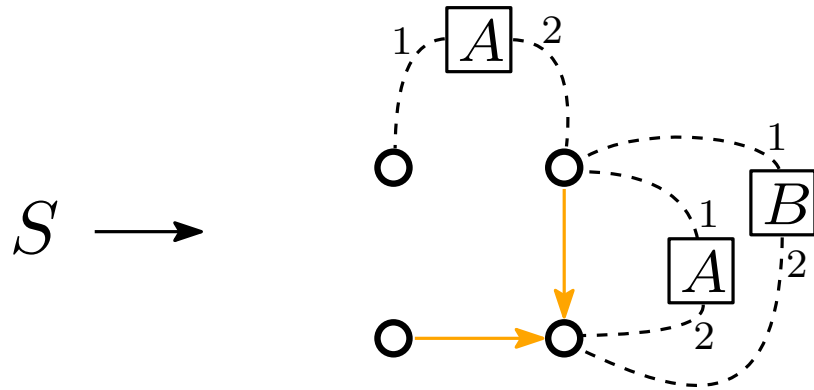


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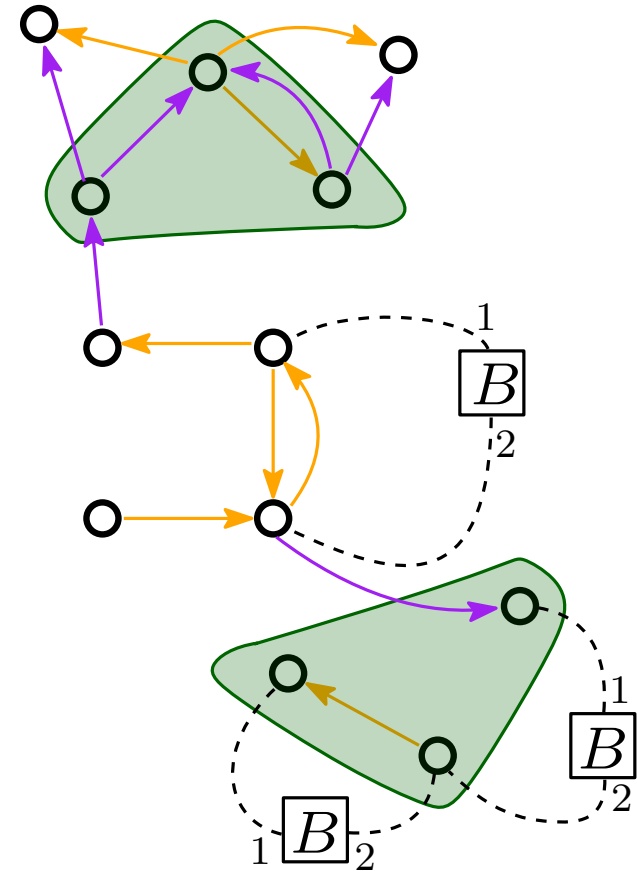


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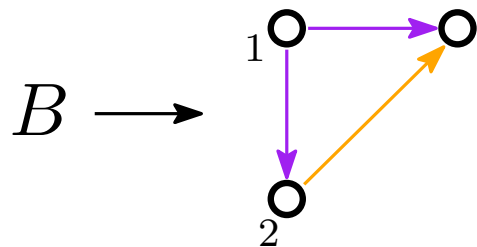
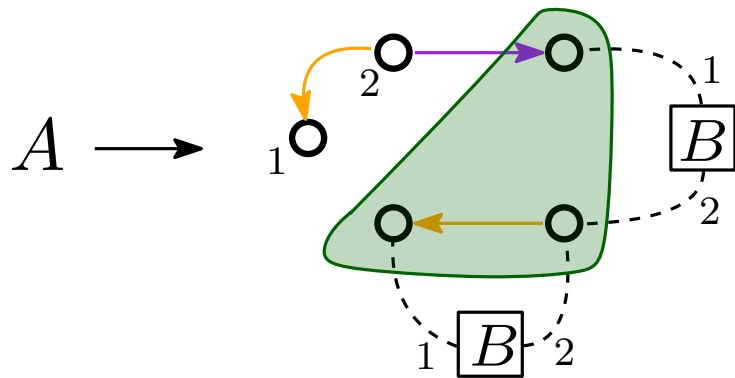
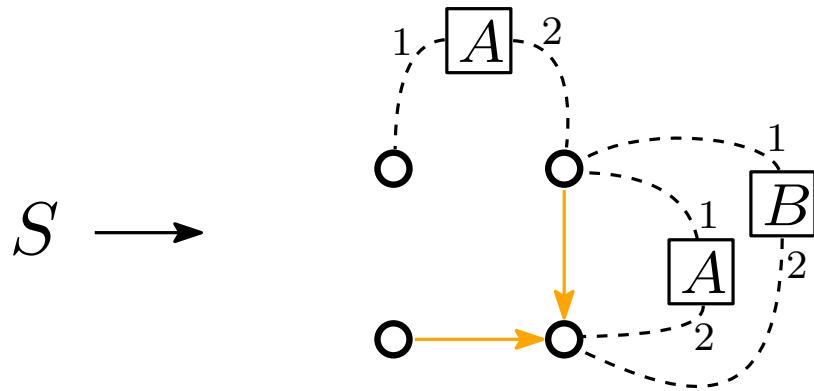


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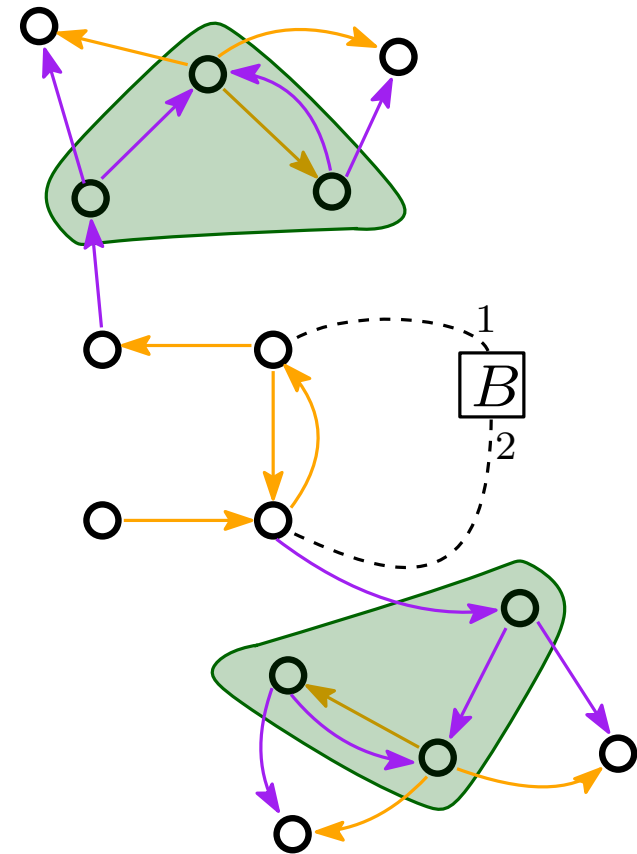


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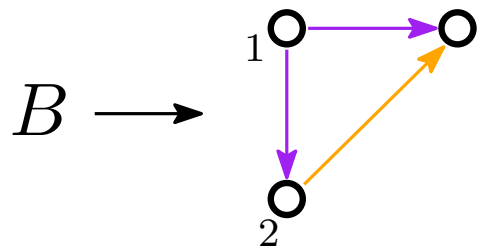
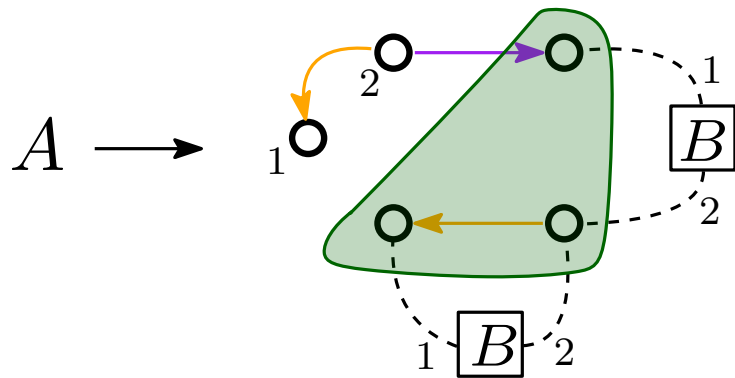
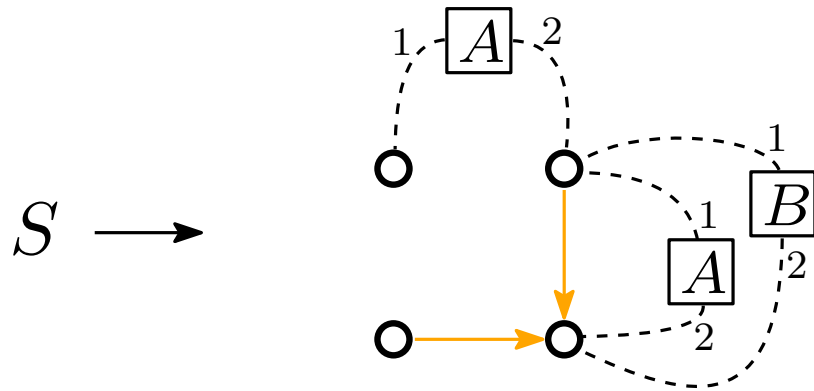


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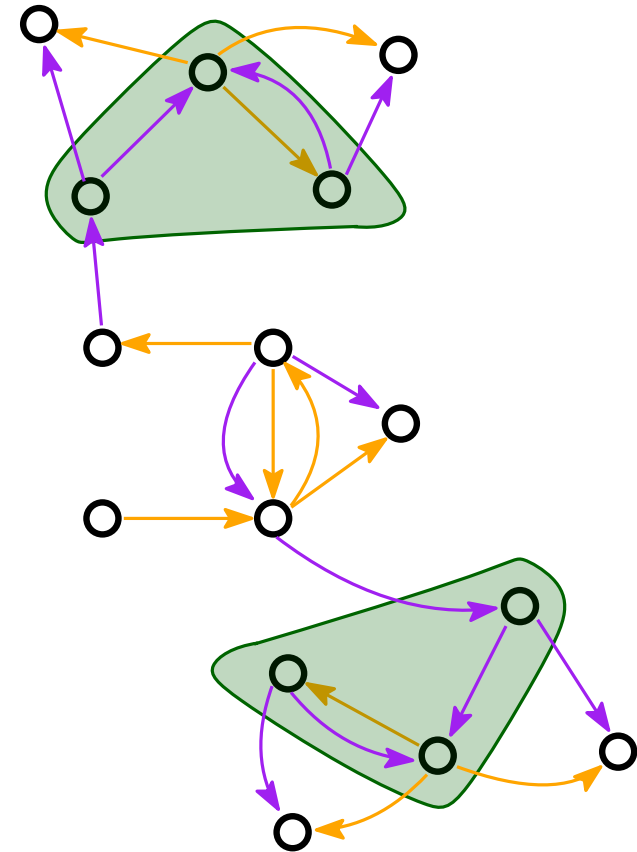


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FO Enumeration Over SLP-Compressed Relational Structures

Theorem:

Given an FO query $q(x_1, x_2, \dots, x_m)$,
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[Durand, Grandjean,
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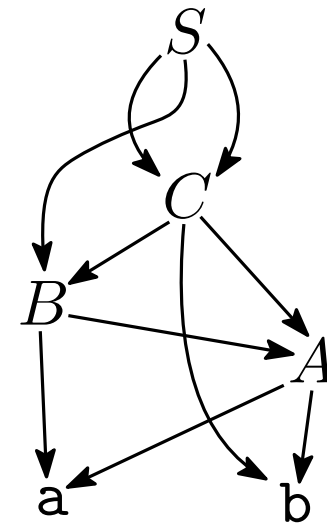
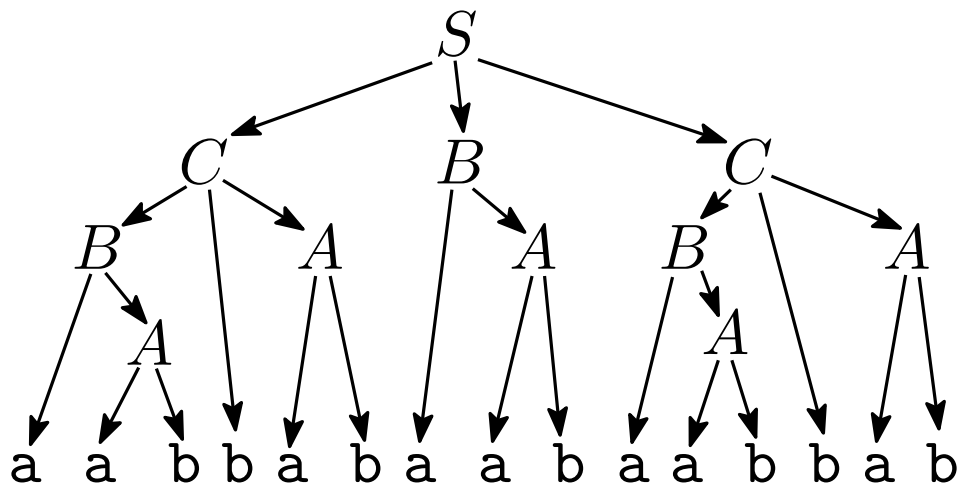
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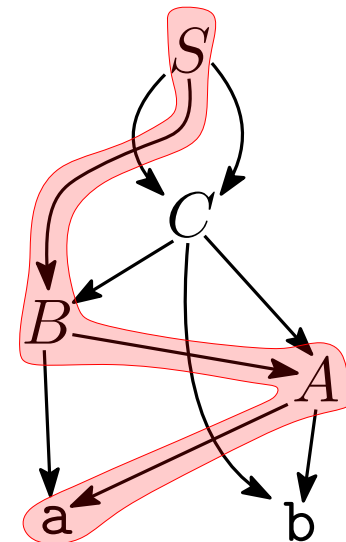
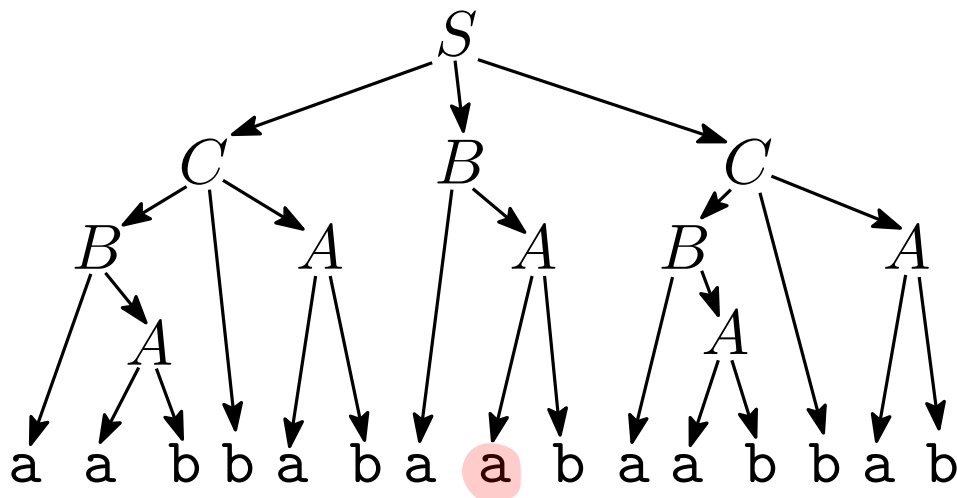
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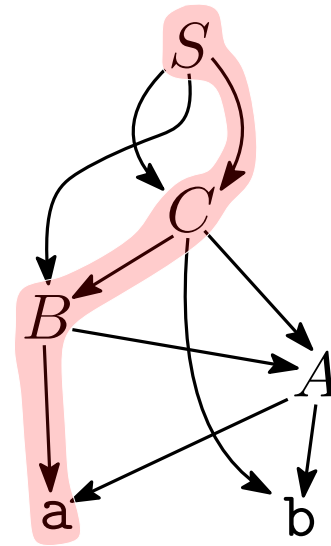
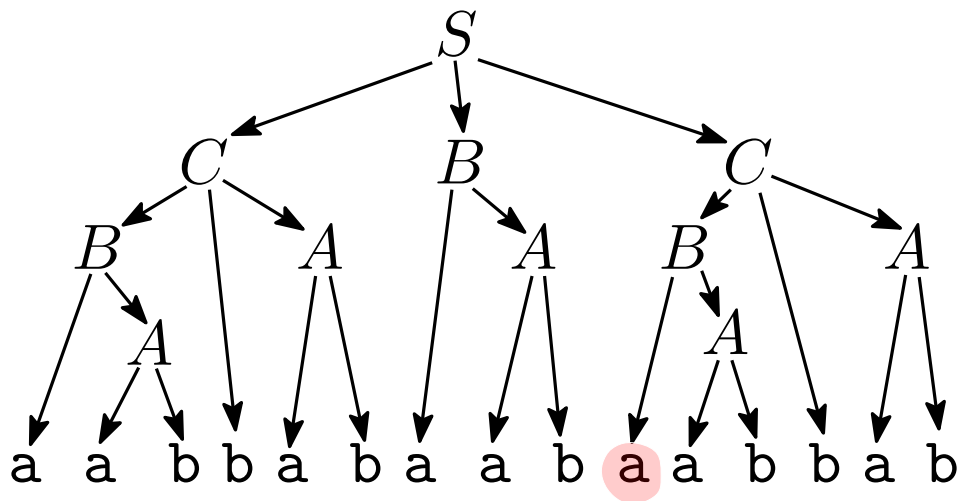
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Thank you very much for your attention!