Enumeration for MSO-Queries on Compressed Trees
Part 2

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Universität Siegen and Humboldt-Universität zu Berlin

DLT 2024

The Main Result

Lohrey, Schmid 2024

Fix a query $\Phi(X)$. One can enumerate $\mathsf{select}(\Phi(X), \mathsf{val}(\mathcal{G}))$ for a given FSLP $\mathcal{G}$ in linear preprocessing time and output-linear delay.

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Proof roadmap:

Step 1: Reduction to a slightly simpler problem about tree automata and DAG-compressed binary trees.

Step 2: Extension of a known enumeration algorithm for tree automata on binary trees to the case of DAG-compressed binary trees,...

Step 3: ...which boils down to solving a problem of enumerating paths in a DAG.

Step 1 – Tree Automata as Query Mechanisms

Let $T = (V, E)$ be a tree and let $S \subseteq V$.

mark(T, S): mark all nodes from S in T.
(formally, label a of node v is replaced by either $(a, 0)$ or $(a, 1)$.)

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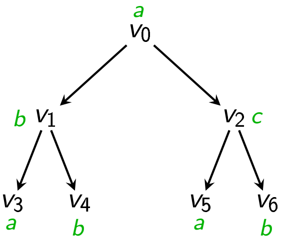
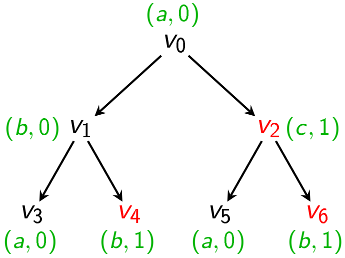


Tree T



mark$(T, \{v_2, v_4, v_6\})$

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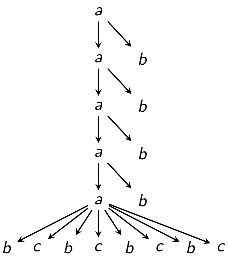
Carme, Niehren, Tommasi, 2004

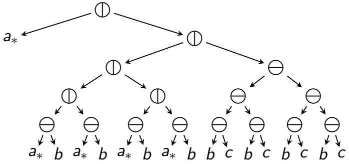
From a given MSO-formula $\Phi(X)$ one can construct a node selecting nondeterministic stepwise tree automaton (nSTA) $\mathcal{A}_\Phi$ such that for every forest F:

$$\text{select}(\mathcal{A}_\Phi, F) = \text{select}(\Phi(X), F)$$

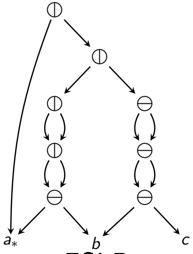
Step 1 – dBUTAs Over DAG-Foldings of Binary Trees



Forest



Forest algebra expression



FSLP

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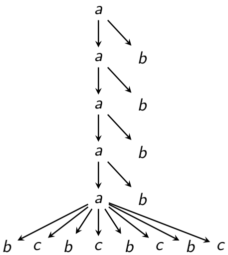

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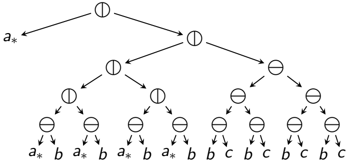

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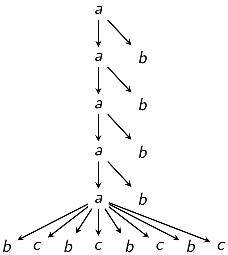

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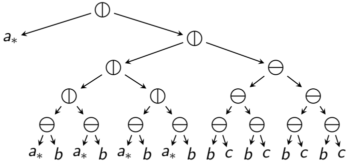

Forest algebra expression

FSLP

Kleest-Meißner, Marasus, Niewerth, 2022

From an nSTA $\mathcal{A}$ working on forests one can construct a deterministic bottom-up tree automaton (dBUTA) $\mathcal{B}$ working on forest algebra expressions with $L(\mathcal{B}) = \{E : \mathrm{val}(E) \in L(\mathcal{A})\}$.

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We have reduced our problem to the following task:

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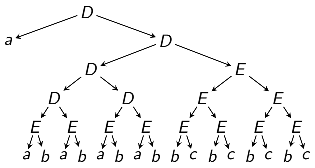
... F is a binary tree...



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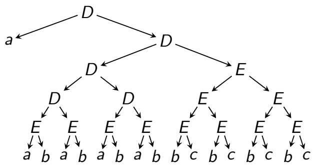
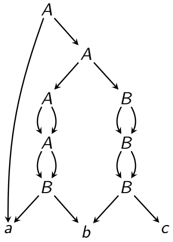


...but given as its DAG folding!



Bagan's Algorithm

For explicit trees, the problem can be solved by Bagan's algorithm:

Theorem — Bagan 2006

For a fixed leaf-selecting dBUTA $\mathcal{B}$ and a binary node-labelled tree T, after a preprocessing in time $O(|T|)$, we can enumerate $\text{select}(\mathcal{B}, T)$ with output linear delay.

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$\rightsquigarrow$ Step 2 – Extend Bagan's algorithm to the DAG-compressed setting

Step 2 – Deterministic Bottom-Up Tree Automata

Leaf-rules: $a \to q$ for label a and state q

Branching-rules: $(r, p, a) \to q$ for label a and states r, p, q

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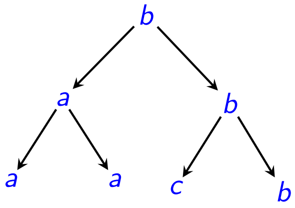

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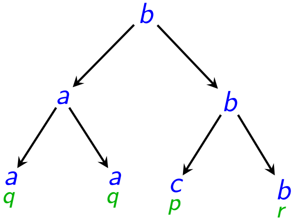

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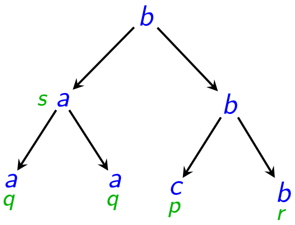

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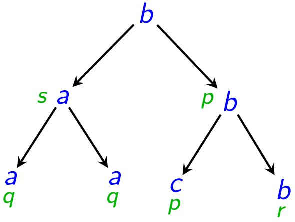

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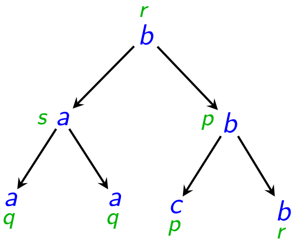

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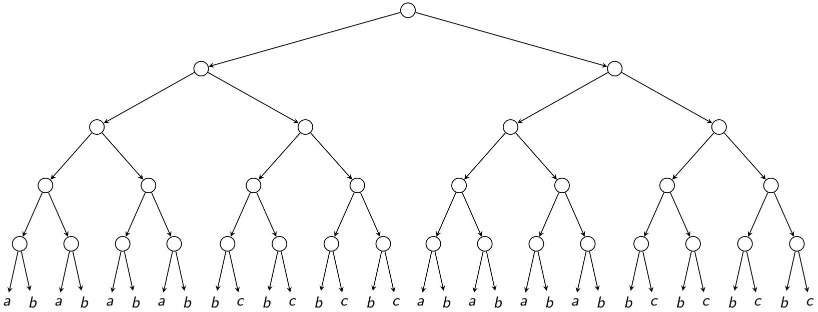
$(s, p, b) \to r$

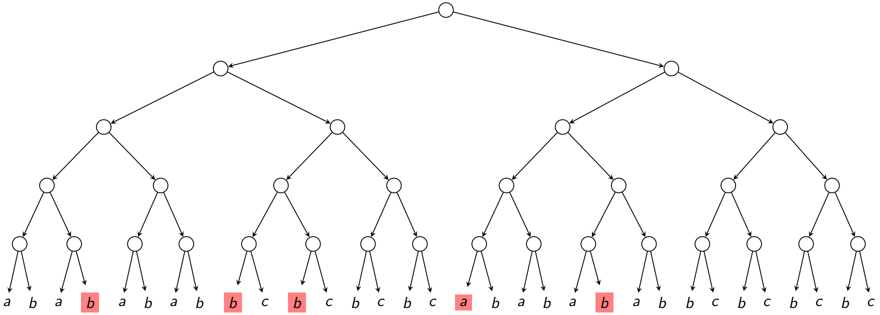
Step 2 – Bagan's Algorithm

Leaf-labelled tree T:



Step 2 – Bagan's Algorithm

Marked tree mark(T, S) for leaf-set S:



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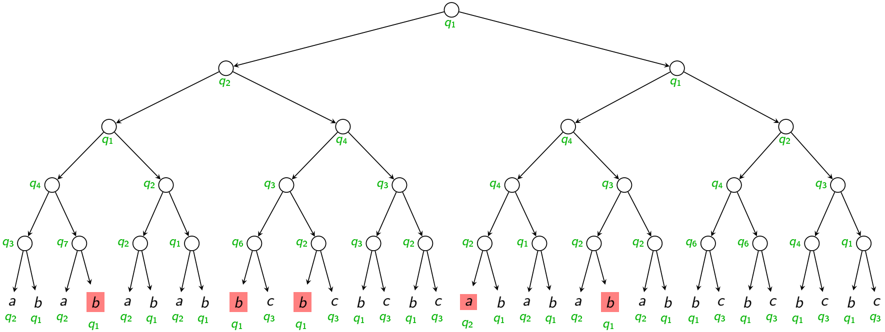
Run on mark(T, S):



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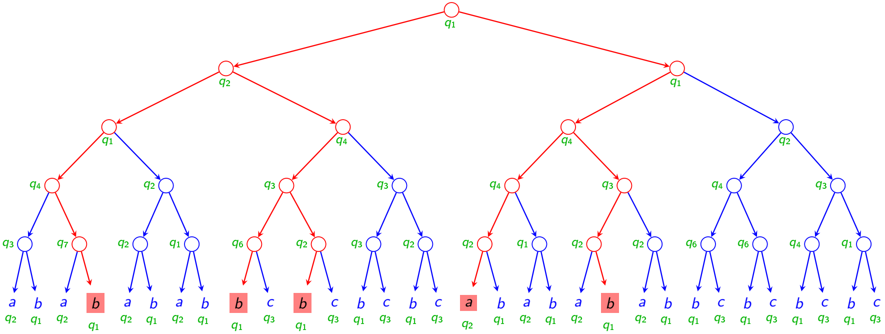

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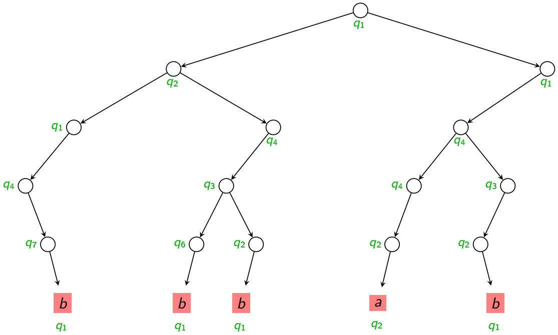

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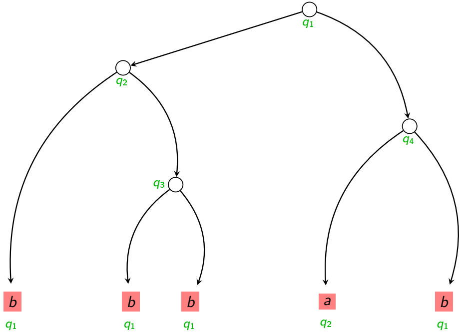

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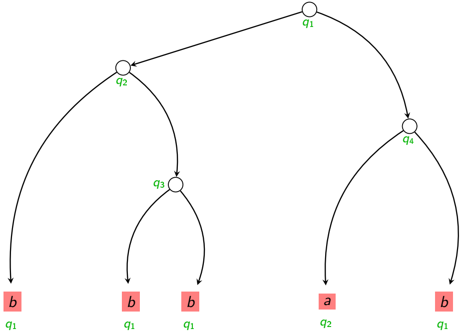
Witness tree for leaf-set S:



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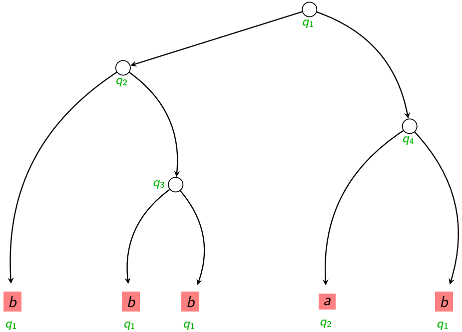

Main idea: Enumerate all witness trees.

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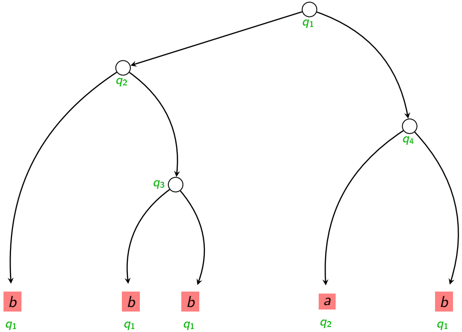

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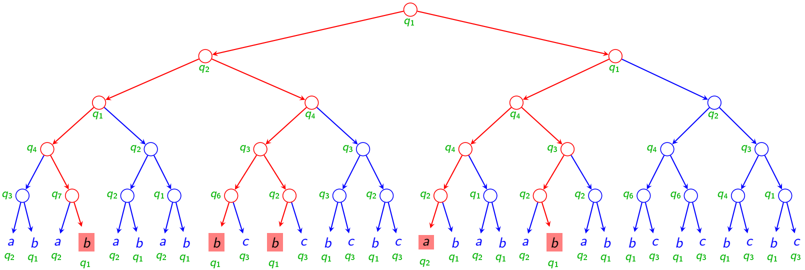
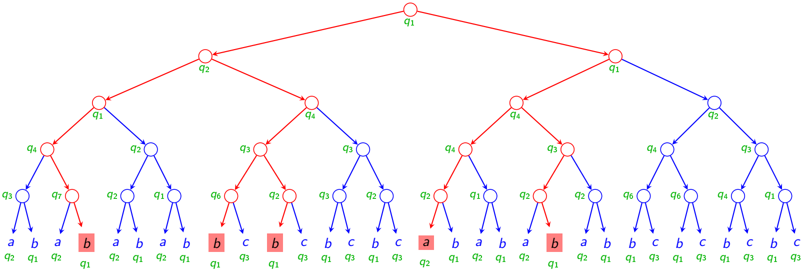
But how to do that?

Step 2 – Bagan's Algorithm



A **configuration** $(v, q) \in V \times Q$ is. . .

Step 2 – Bagan's Algorithm



A **configuration** $(v, q) \in V \times Q$ is...

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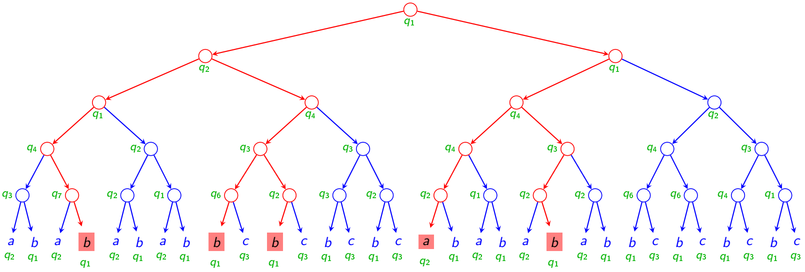

A **configuration** $(v, q) \in V \times Q$ is...

...**active** wrt mark(T, S) if it is red in the run on mark(T, S).

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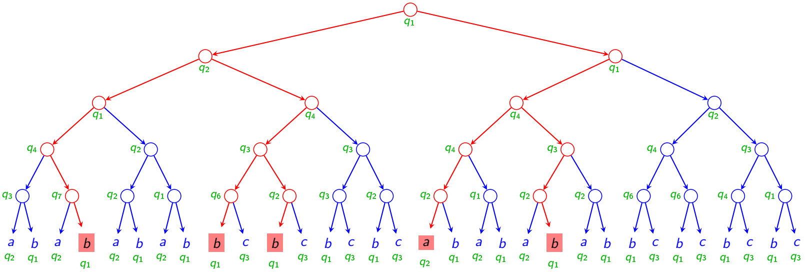

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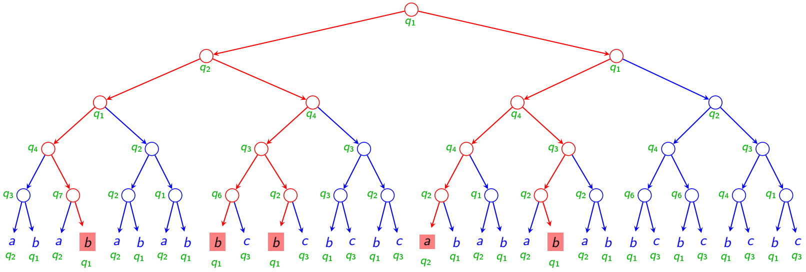

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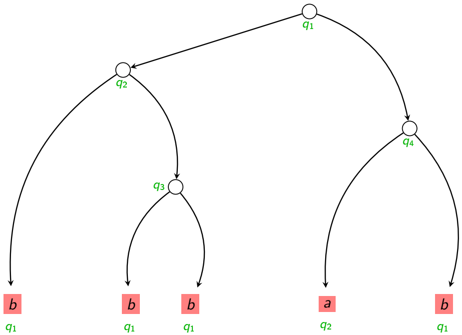
...**nullable** if it is blue in the run on some mark(T, S).

Step 2 – Bagan's Algorithm

Top-down construction of witness trees by appending **useful** configurations:



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Compute a binary relation $\vdash$ on **active** configurations:

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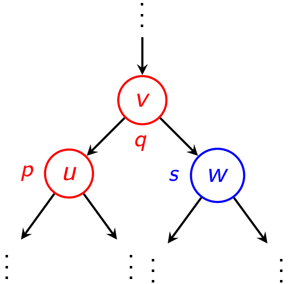

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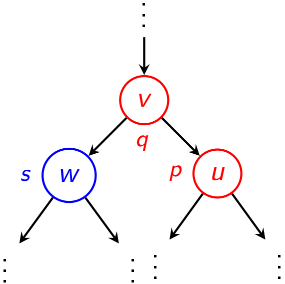

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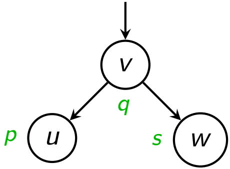

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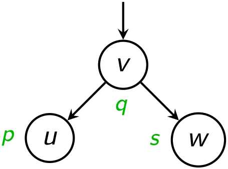

Let (u', p') be useful and $(u, p) \vdash^* (u', p')$.

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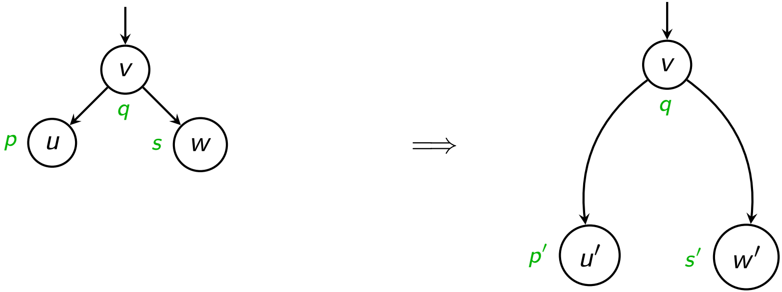

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Goal: Run Bagan's algorithm on the DAG-folding of the tree.

Question: How are tree leaves represented in DAG?

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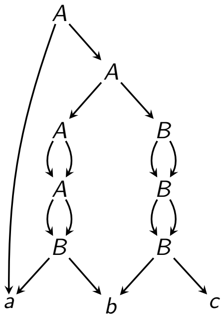

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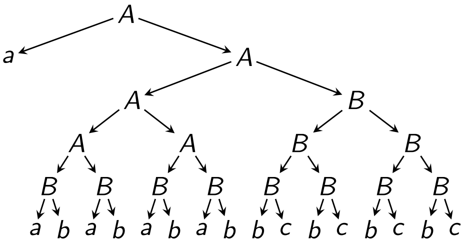

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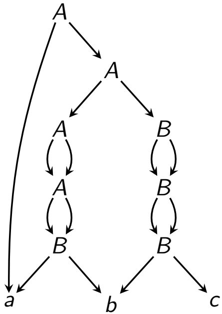

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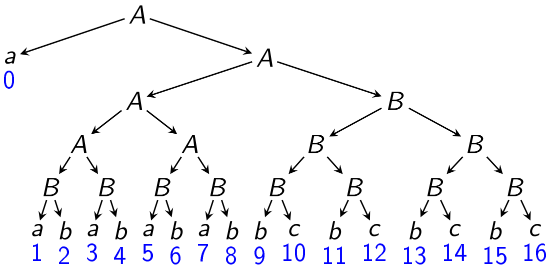

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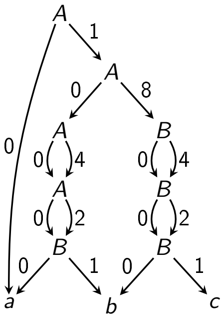

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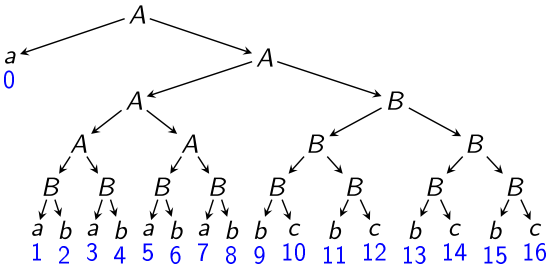

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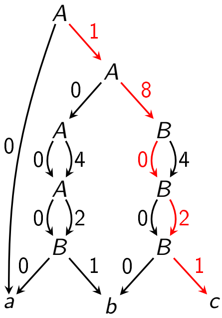

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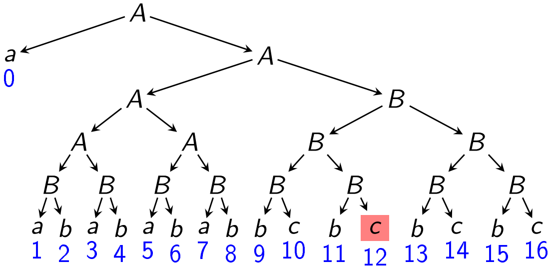

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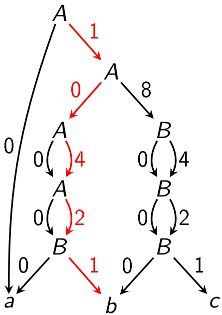
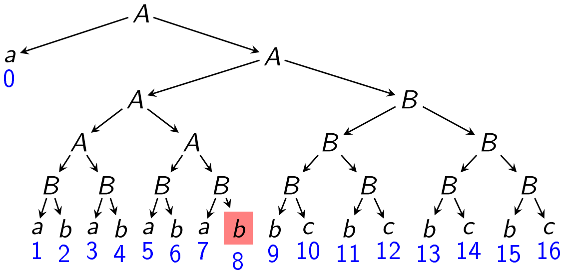


DAG-folding of the tree.



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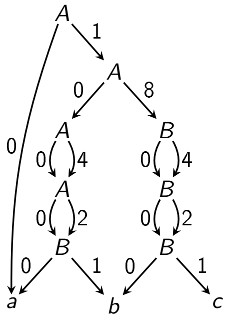

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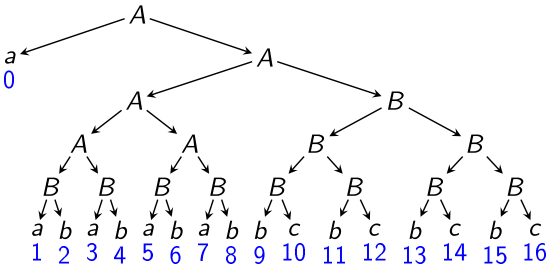

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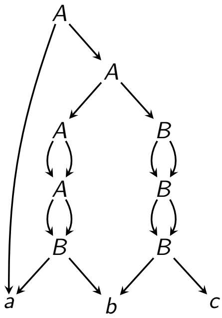

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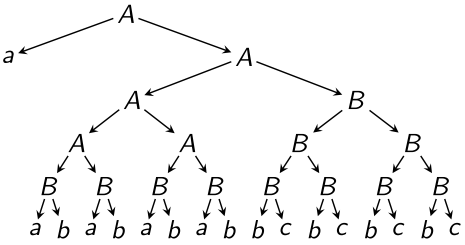

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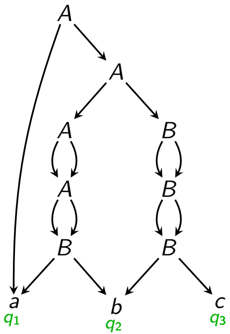

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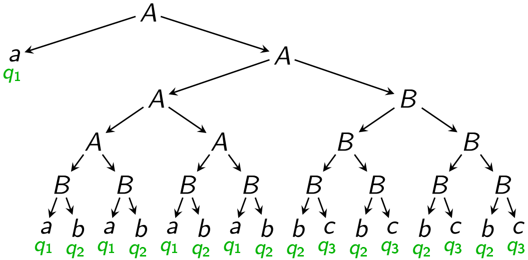

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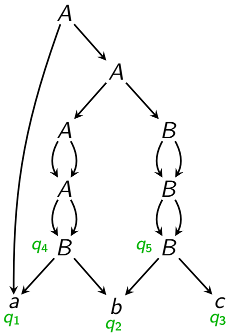
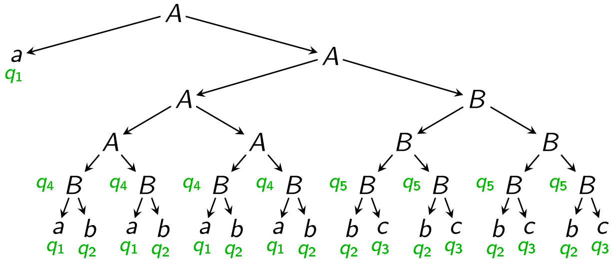


DAG-folding of the tree.



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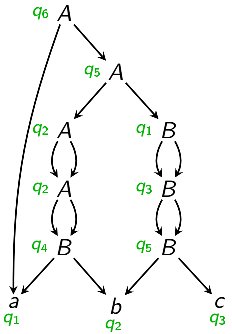

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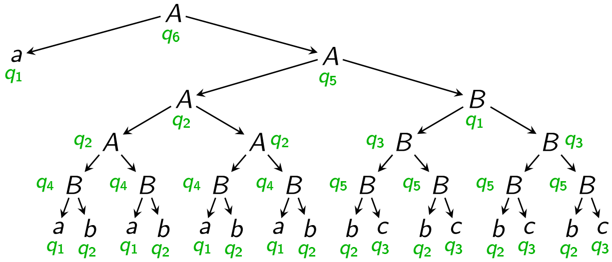

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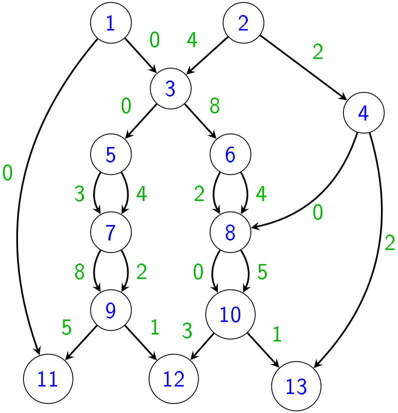
This boils down to the following path enumeration problem in DAGs.

Step 3 – Path Enumeration in DAGs

Let $D = (V, E)$ be a binary DAG with weight function $\gamma : E \to M$.



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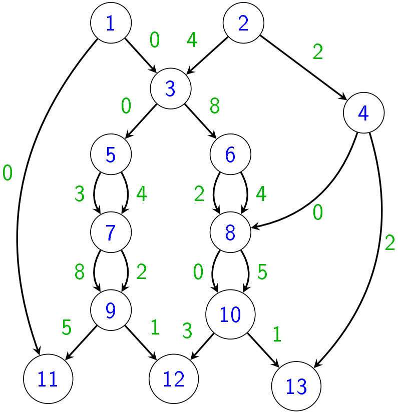

Preprocessing in $O(|D|)$.
For any $s \in V$, enumerate with constant delay all paths π from s to some leaf u represented by $(u, \gamma(\pi))$.

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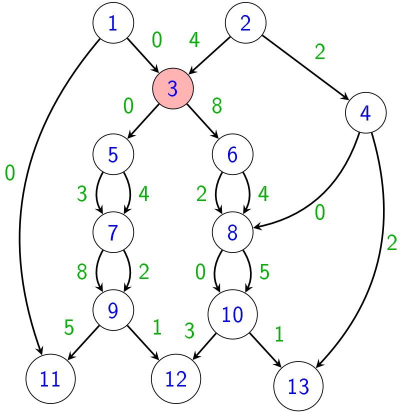

Enumeration for start node 3:

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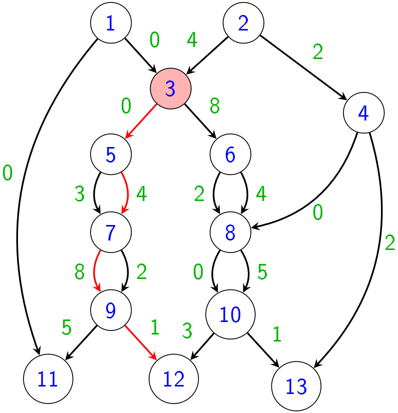

Enumeration for start node 3:
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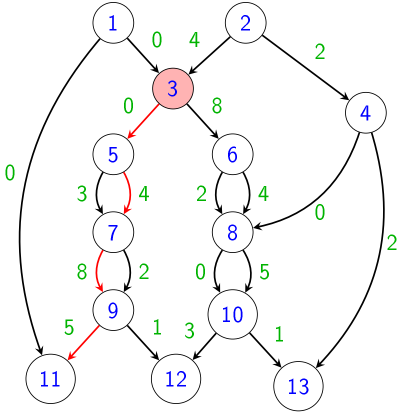

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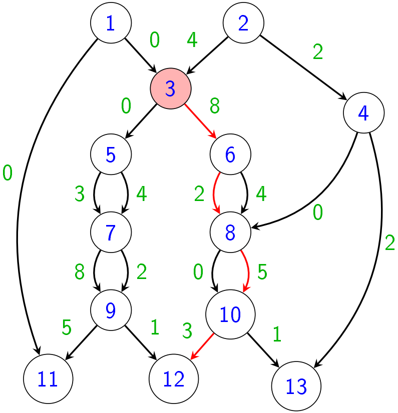

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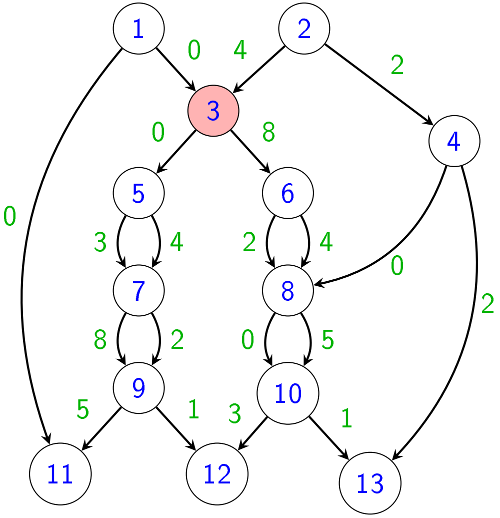
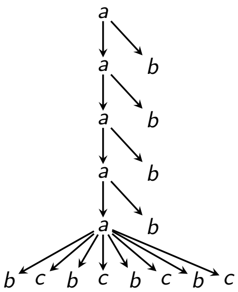
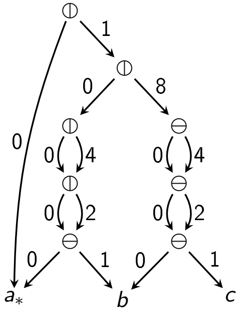


Additional Aspects – Representation of Nodes



Forest



FSLP

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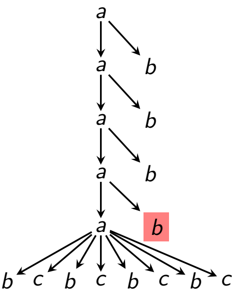

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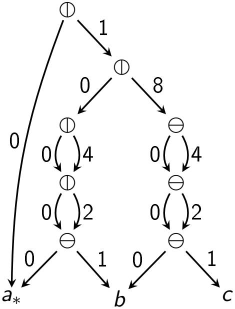

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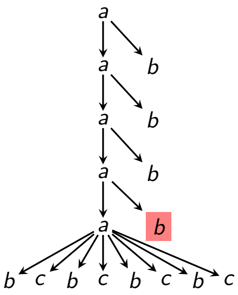

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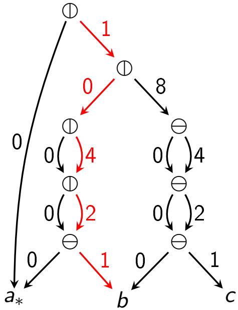

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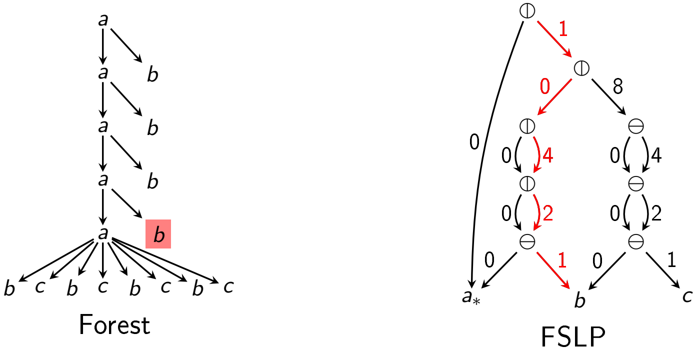

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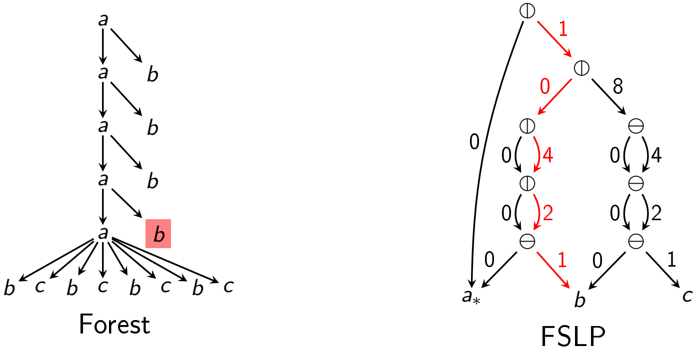

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representation by preorder numbers is also possible (by using edge weights from a complicated monoid).

Additional Aspects – Relabelling Updates

Relabelling updates:

Let F be a forest, v a node of F and x some label.

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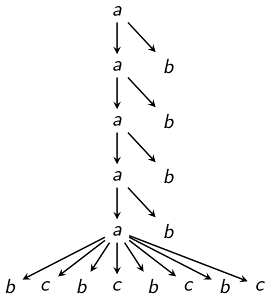

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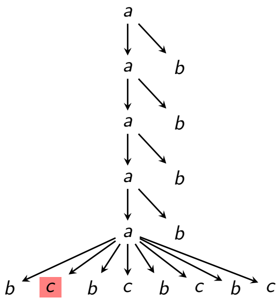

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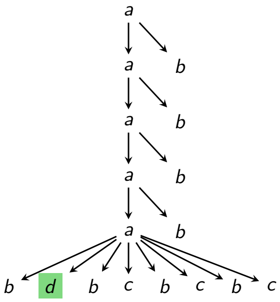

Forest

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Maintaining relabelling updates in the FSLP-compressed setting:

Carry out the linear preprocessing wrt. FSLP $\mathcal{G}$.

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Theorem

We can maintain relabelling updates in the FSLP-compressed setting in time $\mathrm{O}(\log(|F|))$, where the relabelled node is given by its preorder number w.r.t. F.

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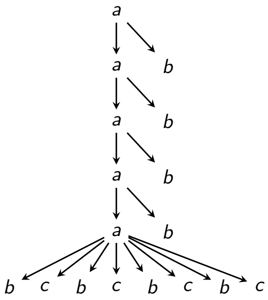

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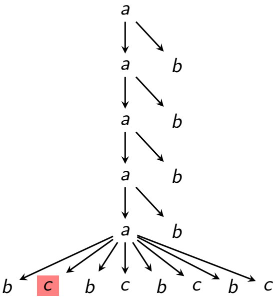

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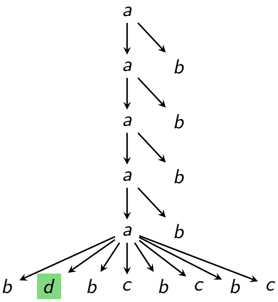

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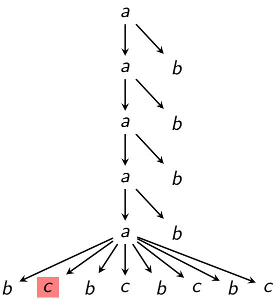

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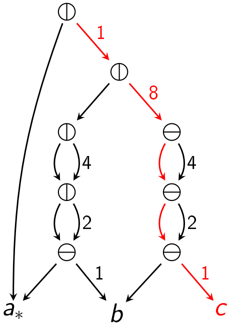

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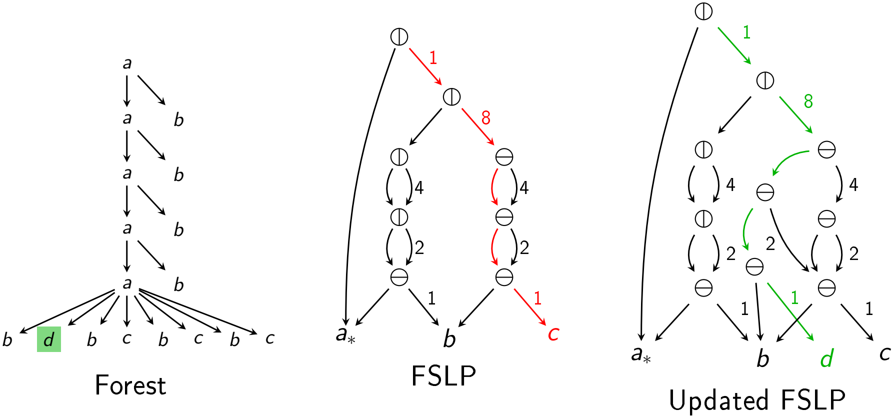

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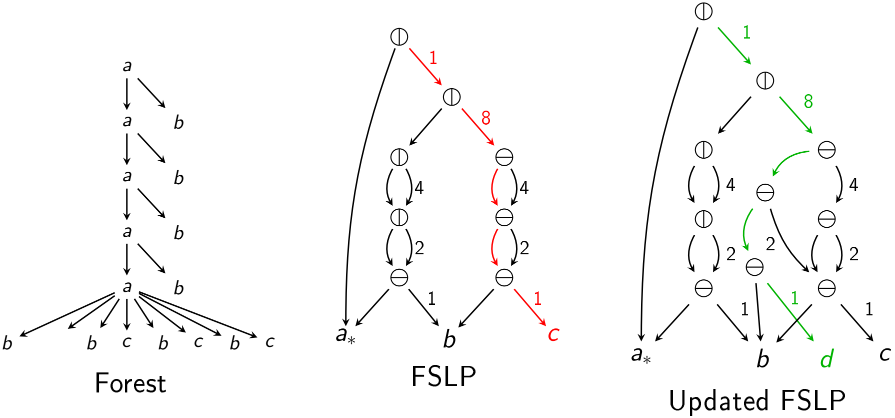

$\rightsquigarrow$ Running time: Height of FSLP.

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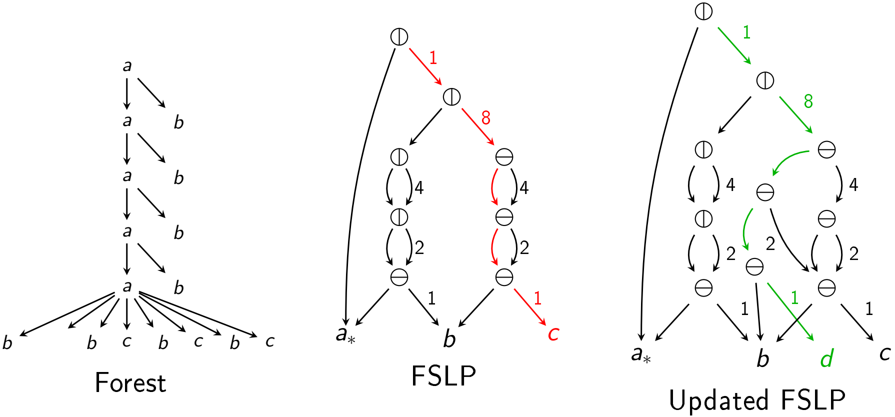

$\rightsquigarrow$ Running time: Height of FSLP.

Height can be bounded by the FSLP balancing theorem:

Theorem (Ganardi, Jez, Lohrey 2021)

FSLPs can be balanced in linear time.

Thank you very much for your attention.